THE APRIL MEETING IN BERKELEY

The three hundred sixty-seventh meeting of the American Mathematical Society was held at the University of California, Berkeley, California, on Saturday, April 6, 1940. The attendance was about eighty including the following forty-one members of the Society:

H. M. Bacon, G. A. Baker, Clifford Bell, B. A. Bernstein, H. F. Blichfeldt, J. L. Brenner, D. G. Chapman, C. L. Clark, P. H. Daus, L. E. Dickson, F. W. Dresch, G. C. Evans, A. L. Foster, E. C. Goldsworthy, E. R. Hedrick, A. E. Ingham, C. G. Jaeger, S. H. Levy, Hans Lewy, A. E. Marston, C. B. Morrey, F. R. Morris, Jerzy Neyman, A. R. Noble, C. A. Noble, C. D. Olds, W. G. Palm, W. T. Puckett, T. M. Putnam, S. E. Rauch, R. M. Robinson, E. B. Roessler, A. C. Schaeffer, W. H. Simons, Gabor Szegö, A. E. Taylor, J. V. Uspensky, F. A. Valentine, W. F. Whitmore. A. R. Williams, O. V. Wood.

General sessions were held both Saturday morning and afternoon. Professors Hedrick, Szegö and Blichfeldt presided over the meeting at various times. The following resolution was passed during the morning session:

"Those in attendance at this meeting of the American Mathematical Society wish to go on record as favoring that a program of mathematics be provided in the secondary schools, beginning normally with algebra in the ninth year, to be available for those who wish to elect it or who otherwise need it in preparation for college work.

"It is felt that a capable student should be able to secure solid geometry and trigonometry in the secondary school."

The titles and cross references to abstracts of papers read at the meeting are given below. Papers whose abstract numbers are followed by the letter *t* were read by title. Professor A. G. Clark and Mr. G. B. Dantzig were introduced by Professor Jerzy Neyman, Mr. J. R. Woolson, Mr. S. P. Avann, Mr. J. P. LaSalle, and Mr. A. B. Mewborn by Professor A. D. Michal, and Mr. R. W. Shephard and Mr. Lewis Nelson by Professor G. C. Evans.

1. J. L. Brenner: *Well-orderable abelian groups whose elements have prime-power order.* (Abstract 46-5-240.)

2. P. H. Daus: *Bisecting circles.* (Abstract 46-5-256.)

3. B. A. Bernstein: *Groups in terms of addition and negation.* (Abstract 46-5-236.)

4. C. D. Olds: *On the number of representations of the square of an integer as the sum of an odd number of squares.* (Abstract 46-5-309.)

5. G. B. Dantzig: *On the non-existence of tests of "Student's" hypothesis, the power function of which would be independent of* σ. (Abstract 46-5-255.)

6. F. W. Dresch: *A mathematical model of a dynamic economic system.* (Abstract 46-5-263.)

7. J. R. Woolson: *The mean of the iteration of linear operators in reflexive Banach spaces.* Preliminary report. (Abstract 46-5-351.)

8. A. C. Schaeffer and Gabor Szegö: *Inequalities for harmonic polynomials in two and three dimensions.* (Abstract 46-5-321.)

9. Lewis Nelson: *An Abel integral equation with constant limits of integration.* (Abstract 46-5-305.)

10. A. R. Williams: *On a certain Cremona transformation between two* $(n-1)$*-spaces in* S_n. (Abstract 46-5-350.)

11. W. T. Puckett: *On arc-preserving transformations.* (Abstract 46-5-316.)

12. R. W. Shephard: *The length of production and related dynamic aspects of a simplified economic system.* Preliminary report. (Abstract 46-5-323.)

13. S. P. Avann: *Lattice automorphisms.* (Abstract 46-5-227-*t*.)

14. J. P. LaSalle: *Applications of the pseudo-norm to the study of linear topological spaces.* (Abstract 46-5-290-*t*.)

15. A. D. Michal and Max Wyman: *Characterization of complex couple spaces.* (Abstract 46-5-299-*t*.)

16. G. B. Dantzig: *Existence of the Neyman-Pearson unbiased test of Type* A. (Abstract 46-5-254-*t*.)

17. A. B. Mewborn: *Abstract local geometry of paths.* II. (Abstract 46-5-297-*t*.)

18. E. T. Bell: *Note on a certain type of diophantine system.* (Abstract 46-5-233-*t*.)

19. E. T. Bell: *Postulational basis for the umbral calculus.* (Abstract 46-5-234-*t*.)

20. A. G. Clark: *Two sample problem concerning Poisson and binomial distributions.* Preliminary report. (Abstract 46-5-249-*t*.)

T. M. PUTNAM,
Associate Secretary

THE APRIL MEETING IN CHICAGO

The three hundred sixty-eighth meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 12–13, 1940. Two hundred twelve persons registered including the following one hundred eighty-one members of the Society:

A. A. Albert, G. E. Albert, P. H. Anderson, T. W. Anderson, Max Astrachan, J. V. Atanasoff, W. L. Ayres, Reinhold Baer, R. H. Bardell, R. W. Barnard, R. C. F. Bartels, R. A. Beaumont, J. H. Bell, H. R. Beveridge, S. F. Bibb, G. A. Bliss, Morris Bloom, Henry Blumberg, L. M. Blumenthal, F. L. Brooks, R. S. Burington, H. E. Burns, L. E. Bush, W. H. Bussey, J. W. Calkin, C. C. Camp, H. H. Campaigne, T. E. Caywood, E. W. Chittenden, R. V. Churchill, M. D. Clement, R. H. Cole, B. H. Colvin, Max Coral, J. J. Corliss, E. L. Crow, D. R. Curtiss, J. E. Davis, W. M. Davis, A. H. Diamond, J. M. Dobbie, J. L. Doob, D. M. Dribin, R. J. Duffin, Ben Dushnik, J. J. Eachus, Samuel Eilenberg, W. S. Erickson, E. B. Escott, H. P. Evans, H. S. Everett, L. R. Ford, Bernard Friedman, D. G. Fulton, H. L. Garabedian, E. R. Garbe, R. E. Gaskell, B. H. Gere, F. J. Gerst, B. C. Getchell, H. A. Giddings, E. L. Godfrey, H. H. Goldstine, G. D. Gore, Cornelius Gouwens, L. M. Graves, J. W. Green, L. W. Griffiths, V. G. Grove, O. H. Hamilton, P. C. Hammer, M. L. Hartung, G. E. Hay, K. E. Hazard, O. C. Hazlett, A. E. Heins, E. D. Hellinger, M. R. Hestenes, J. F. Heyda, T. H. Hildebrandt, J. J. L. Hinrichsen, D. L. Holl, A. S. Householder, D. H. Hyers, M. H. Ingraham, S. B. Jackson, Fritz John, Chosaburo Kato, J. F. Kenney, W. C. Krathwohl, A. C. Ladner, R. E. Langer, Lincoln LaPaz, D. H. Leavens, R. A. Leibler, J. H. Levin, P. E. Lewis, A. O. Lindstrum, M. I. Logsdon, A. T. Lonseth, W. C. McDaniel, C. C. MacDuffee, H. M. MacNeille, Morris Marden, J. R. Mayor, Karl Menger, H. L. Meyer, A. N. Milgram, D. D. Miller, W. E. Milne, W. L. Mitchell, M. G. Moore, R. L. Moore, C. W. Moran, E. J. Moulton, A. L. Nelson, H. E. Newell, K. L. Nielsen, I. M. Niven, E. A. Nordhaus, M. A. Norval, E. N. Oberg, J. W. Odle, Rufus Oldenburger, E. W. Paxson, J. F. Paydon, E. R. Peck, G. H. Peebles, I. E. Perlin, Sam Perlis, G. B. Price, Tibor Radó, E. D. Rainville, R. B. Rasmusen, W. T. Reid, C. E. Rickart, R. F. Rinehart, W. E. Roth, E. H. Rothe, M. A. Sadowsky, R. G. Sanger, W. T. Scott, C. E. Sealander, M. E. Shanks, C. G. Shover, H. A. Simmons, M. M. Slotnick, F. C. Smith, G. G. Speeker, D. W. Starr, N. E. Steenrod, B. M. Stewart, H. P. Thielman, R. M. Thrall, Gerhard Tintner, L. W. Tordella, Leonard Tornheim, H. C. Trimble, W. J. Trjitzinsky, P. L. Trump, J. W. Tukey, V. J. Varino, J. I. Vass, H. E. Vaughan, H. S. Wall, Henry Wallman, T. O. Walton, L. E. Ward, K. W. Wegner, M. J. Weiss, C. P. Wells, M. E. Wescott, G. W. Whaples, G. W. Whitehead, L. R. Wilcox, K. P. Williams, L. A. Wolf, M. C. Wolf, Y. K. Wong, F. E. Wood, J. W. T. Youngs.

On Friday morning the meeting was held in three sections, Analysis, Professor R. E. Langer presiding; Geometry and Algebra, Professor Reinhold Baer presiding; and Topology and the Theory of Sets, Professor N. E. Steenrod presiding. On Friday afternoon Professor W. J. Trjitzinsky gave the Symposium Lecture entitled *General the-*

ory of functions of a complex variable. Professor E. W. Chittenden presided at this lecture. On Saturday morning there were two sections, Analysis with Professor W. T. Reid presiding and Algebra with Professor C. C. MacDuffee presiding.

All sessions were held in Eckhart Hall and the adjoining Ryerson Laboratory.

On Friday afternoon a tea was given by the ladies of the Department of Mathematics of the University of Chicago in the Commons Room of Eckhart Hall.

On Friday evening a dinner was held at Burton Court for members and guests with an attendance of one hundred sixty-eight. Professor H. S. Everett of the University of Chicago acted as toastmaster, and brief talks were given by Professors W. H. Bussey, Marie J. Weiss, G. B. Price, L. R. Ford, and R. L. Moore.

Titles and cross references to the abstracts of the papers read at this meeting follow below. Papers numbered 1 to 9 were read before the Analysis section Friday morning, papers 10 to 16 before the section for Geometry and Algebra, papers 17 to 24 before the section for Topology and the Theory of Sets, papers 25 to 32 before the Analysis section Saturday morning, and papers 33 to 39 before the Algebra section Saturday morning. Papers 40 to 61, whose abstract numbers are followed by the letter *t*, were read by title. Mr. F. P. Jenks and Mr. J. C. Abbott were introduced by Professor Karl Menger, Dr. W. S. Snyder by Professor Tibor Radó, Mr. H. J. Miser by Professor L. R. Ford, and Mr. Roy Dubisch by Professor A. A. Albert. Paper number 2 was read by Dr. Duffin, 3 by Dr. Bartels, 4 by Dr. Leibler, 18 by Professor Dushnik, 29 by Dr. Reichelderfer, and 37 by Mr. Porges.

1. R. E. Gaskell: *A problem in heat conduction and an expansion theorem.* (Abstract 46-5-271.)

2. R. J. Duffin and J. J. Eachus: *The converse of a closure theorem of Paley and Wiener.* (Abstract 46-5-265.)

3. R. C. F. Bartels and R. V. Churchill: *An extension of Duhamel's theorem.* (Abstract 46-5-230.)

4. J. L. Doob and R. A. Leibler: *On the spectral analysis of a certain transformation.* Preliminary report. (Abstract 46-5-261.)

5. Fritz John: *The Dirichlet problem for a hyperbolic equation.* (Abstract 46-5-284.)

6. R. V. Churchill: *Termwise integration of Sturm-Liouville expansions.* Preliminary report. (Abstract 46-5-247.)

7. A. E. Heins: *On the solution of partial difference equations: symmetric boundary conditions.* (Abstract 46-5-277.)

8. K. L. Nielsen: *Concerning general boundary value problems for linear differential equations.* Preliminary report. (Abstract 46-5-307.)

9. Gerhard Tintner: *Fourier integrals as Bessel differential operators.* Preliminary report. (Abstract 46-5-333.)

10. F. P. Jenks: *Order and parallelism in the non-euclidean geometry of joining and intersecting.* (Abstract 46-5-283.)

11. J. C. Abbott: *Projective theory of congruence in non-euclidean geometry.* Preliminary report. (Abstract 46-5-225.)

12. L. M. Blumenthal: *A new concept in distance geometry, with applications to spherical subsets.* (Abstract 46-7-354.)

13. Sam Perlis: *Scalar extensions of algebras with exponent equal to index.* (Abstract 46-5-315.)

14. D. M. Dribin: *The norm residue symbol for solvable algebraic number fields.* (Abstract 46-5-264.)

15. W. L. Mitchell: *Topological rings and infinite matrices.* (Abstract 46-5-302.)

16. V. J. Varino: *A note on matrices over a principal ideal ring.* (Abstract 46-5-335.)

17. E. W. Paxson: *Strictly convex metric spaces.* (Abstract 46-5-312.)

18. Ben Dushnik and E. W. Miller: *On partially ordered sets.* (Abstract 46-5-266.)

19. A. N. Milgram: *Partially ordered sets and the covering theorems of topology.* (Abstract 46-5-301.)

20. Samuel Eilenberg: *Ordered topological spaces.* (Abstract 46-5-268.)

21. G. E. Albert: *On contiguous point spaces and their applications.* (Abstract 46-3-138.)

22. D. D. Miller: *Hereditary properties of continuous transformations.* Preliminary report. (Abstract 46-5-211.)

23. J. W. T. Youngs: *On parametric representations of surfaces.* Preliminary report. (Abstract 46-5-352.)

24. R. L. Moore: *Concerning separability.* (Abstract 46-5-303.)

25. H. P. Thielman: *On the convex solution of a certain functional equation.* (Abstract 46-5-331.)

26. H. H. Goldstine: *Linear functionals and integrals in abstract spaces.* (Abstract 46-5-273.)

27. Tibor Radó: *On a lemma of McShane.* (Abstract 46-3-150.)

28. Karl Menger: *On Green's formula and the integral of derivatives.* (Abstract 46-5-210.)

29. Tibor Radó and P. V. Reichelderfer: *Note on an inequality of Steiner.* (Abstract 46-3-184.)

30. W. S. Snyder: *Functions of simple figures.* Preliminary report. (Abstract 46-5-325.)

31. J. W. Calkin: *A quotient ring over the ring of bounded operators in Hilbert space.* I. (Abstract 46-5-203.)

32. H. J. Miser: *Regions and their "patterns" in conformal mapping.* (Abstract 46-7-365.)

33. R. A. Beaumont: *Projections of non-abelian groups upon abelian groups.* Preliminary report. (Abstract 46-5-232.)

34. R. M. Thrall: *A note on a theorem by Witt.* (Abstract 46-5-332.)

35. P. E. Lewis: *Characters of abelian groups.* (Abstract 46-5-291.)

36. H. H. Campaigne: *Multiplication systems.* Preliminary report. (Abstract 46-5-242.)

37. Rufus Oldenburger and A. E. Porges: *The minimal numbers of binary forms.* (Abstract 46-5-212.)

38. Leonard Tornheim: *Linear forms in function fields.* (Abstract 46-3-196.)

39. Roy Dubisch: *Non-cyclic algebras of degree four and exponent two with pure maximal subfields.* (Abstract 46-5-206.)

40. H. L. Garabedian and H. S. Wall: *Hausdorff matrices and continued fractions.* (Abstract 46-3-171-*t*.)

41. G. E. Albert: *On quasi-metric spaces.* Preliminary report. (Abstract 46-3-154-*t*.)

42. Y. K. Wong: *On non-modular matrices.* (Abstract 46-5-224-*t*.)

43. Y. K. Wong: *Interchange of J-processes in bilinear forms involving non-modular matrices.* (Abstract 46-5-223-*t*.)

44. Gordon Pall: *On the arithmetic of quaternions.* (Abstract 46-5-215-*t*.)

45. J. W. Calkin: *A generalization of a theorem of Weyl.* (Abstract 46-5-204-*t*.)

46. J. W. Calkin: *Functions of several variables and absolute continuity.* I. (Abstract 46-5-205-*t*.)

47. Rufus Oldenburger: *On a class of non-negative matrices.* (Abstract 46-5-214-*t*.)

48. Rufus Oldenburger: *Polynomials in several variables.* (Abstract 46-5-213-*t*.)

49. M. S. Webster: *Maximum of certain fundamental Lagrange interpolation polynomials.* (Abstract 46-5-342-*t*.)

50. C. W. Vickery: *On spaces* ($\mathcal{E}$) *and Moore spaces.* (Abstract 46-5-337-*t*.)

51. Reinhold Baer: *Sylow theorems for infinite groups.* (Abstract 46-5-229-*t*.)

52. Reinhold Baer: *Nets and groups.* II. (Abstract 46-5-228-*t*.)

53. Karl Menger: *On shortest polygonal approximations to a curve.* (Abstract 46-5-296-*t*.)

54. A. N. Milgram: *On the length of continuous curves.* (Abstract 46-5-300-*t*.)

55. Samuel Eilenberg: *On spherical cycles.* (Abstract 46-5-267-*t*.)

56. W. H. Ingram: *A generalization of Erhard Schmidt's solution of the nonhomogeneous integral equation.* Preliminary report. (Abstract 46-5-282-*t*.)

57. I. E. Perlin: *Sufficient conditions that polynomials in several variables be positive.* (Abstract 46-5-314-*t*.)

58. H. H. Campaigne: *A lower limit on the number of hypergroups of a given order.* (Abstract 46-5-241-*t*.)

59. J. M. Feld: *Whirl-similitudes, euclidean kinematics and non-euclidean geometry.* Preliminary report. (Abstract 46-5-270-*t*.)

60. R. V. Churchill: *A problem in the conduction of heat.* (Abstract 46-5-246-*t*.)

61. R. L. Wilder: *Characterization of the lower dimensional generalized manifolds by positional properties in S_n.* (Abstract 46-5-349-*t*.)

W. L. AYRES,
Associate Secretary

THE APRIL MEETING IN WASHINGTON

The three hundred sixty-ninth meeting of the American Mathematical Society was held at The George Washington University, Washington, D. C., on Friday and Saturday, April 26–27, 1940. The attendance included the following one hundred sixty members of the Society:

C. R. Adams, C. B. Allendoerfer, R. C. Archibald, J. V. Atanasoff, Harry Bateman, E. E. Betz, Archie Blake, R. P. Boas, A. T. Brauer, Richard Brauer, R. S. Burington, Herbert Busemann, W. E. Byrne, S. S. Cairns, J. W. Calkin, W. B. Campbell, C. E. Carey, Randolph Church, Paul Civin, J. M. Clarkson, G. R. Clements, Abraham Cohen, I. S. Cohen, Nancy Cole, M. J. Cox, H. S. M. Coxeter, P. D. Crout, H. B. Curry, Tobias Dantzig, C. H. Dowker, Arnold Dresden, J. E. Eaton, Samuel Eilenberg, L. P. Eisenhart, M. L. Elveback, Paul Erdös, W. K. Feller, E. J. Finan, D. A. Flanders, W. W. Flexner, R. M. Foster, Hilda Geiringer, Abe Gelbart, P. W. Gilbert, Wallace Givens, Michael Goldberg, Michael Golomb, B. L. Hagen, D. W. Hall, O. G. Harrold, Philip Hartman, G. A. Hedlund, Edward Helly, Olaf Helmer, J. G. Herriot, Einar Hille, M. P. Hollcroft, T. R. Hollcroft, Witold Hurewicz, Nathan Jacobson, S. A. Jennings, Evan Johnson, R. F. Johnson, F. E. Johnston, H. A. Jordan, E. R. van Kampen, Wilfred Kaplan, J. L. Kelley, S. C. Kleene, J. R. Kline, E. R. Kolchin, H. L. Krall, W. D. Lambert, O. E. Lancaster, A. E. Landry, Solomon Lefschetz, D. T. McClay, N. H. McCoy, Brockway McMillan, E. J. McShane, H. M. MacNeille, Dorothy Manning, A. V. Martin, M. H. Martin, F. M. Mears, A. D. Michal, Don Mittleman, T. W. Moore, R. C. Morrow, Marston Morse, D. C. Murdoch, F. D. Murnaghan, F. G. Myers, John von Neumann, C. O. Oakley, Oystein Ore, F. W. Owens, H. B. Owens, J. C. Oxtoby, N. G. Parke, G. W. Patterson, P. M. Pepper, R. S. Phillips, H. A. Rademacher, O. J. Ramler, H. J. Riblet, J. N. Rice, R. G. D. Richardson, J. F. Ritt, J. H. Roberts, H. P. Robertson, G. de B. Robinson, L. B. Robinson, R. E. Root, J. E. Rosenthal, Barkley Rosser, O. F. G. Schilling, Abraham Schwartz, G. E. Schweigert, C. H. W. Sedgewick, I. E. Segal, Wladimir Seidel, Abraham Seidenberg, L. W. Sheridan, R. G. Simond, Abraham Sinkov, H. L. Smith, J. P. Smith, P. A. Smith, Virgil Snyder, F. W. Sohon, V. E. Spencer, J. M. Stetson, E. B. Stouffer, J. L. Synge, J. H. Taylor, J. M. Thomas, E. W. Titt, C. B. Tompkins, A. W. Tucker, L. B. Tuckerman, J. W. Tukey, Oswald Veblen, G. L. Walker, A. D. Wallace, J. V. Wehausen, F. M. Weida, Albert Wertheimer, F. J. Weyl, Hermann Weyl, A. P. Wheeler, C. H. Wheeler, P. A. White, A. L. Whiteman, G. T. Whyburn, S. S. Wilks, John Williamson, H. A. Wood, J. W. Wrench, J. W. T. Youngs.

The meeting opened Friday morning in two sections, Analysis, Professor J. L. Synge presiding, and Topology, Professor A. D. Michal presiding.

At the general session Friday afternoon, Vice President F. D. Murnaghan presiding, Professor A. W. Tucker of Princeton University gave an address entitled *Complexes and tensors*. An informal meeting of those especially interested in the application of tensor

analysis to electrical engineering was held immediately following this address.

The general session was followed by two sections: Algebra, Professor Hermann Weyl presiding; Applied Mathematics, Geometry and Logic, Professor S. C. Kleene presiding.

Two general sessions were held Saturday morning. At the first, Professor Arnold Dresden presiding, Professor Wladimir Seidel of the University of Rochester gave an address entitled *Univalent and multivalent functions*. The second, Vice President C. R. Adams presiding, was for short papers on various subjects.

All sessions were held in the Hall of Government, The George Washington University.

On Friday afternoon, tea was served in Columbian House, Mrs. J. H. Taylor, hostess.

The dinner Friday evening at the Cosmos Club was attended by one hundred thirty-eight mathematicians and guests. The toastmaster, Professor Marston Morse, introduced President C. H. Marvin of The George Washington University who welcomed the mathematicians and generously invited the Society to hold its meetings at The George Washington University whenever it chose to do so. President Marvin was followed by three speakers, Professors Oswald Veblen, Oystein Ore and E. J. McShane, who spoke briefly on *The Mathematical Reviews*, *Art and mathematics*, and *Mathematics in South America*, respectively.

The Council met at 9:00 P.M., Friday, April 26, in the Holmes Room of the Cosmos Club.

At the general session Saturday morning Professor Barkley Rosser presented a resolution expressing the appreciation and thanks of the Society to the President of The George Washington University, the members of the local committee, the Mathematics Department and all others concerned for their competent arrangements and cordial hospitality. This resolution was unanimously adopted.

The Secretary announced the election of the following seventy-three persons to membership in the Society:

Mr. Lucius Anthony, Seattle, Wash.;
Professor José Barral-Souto, University of Buenos Aires;
Dr. Alfred Basch, Holy Cross College, Worcester, Mass.;
Mr. Jacob Bearman, University of Minnesota;
Professor Richard A. Beth, Michigan State College;
Dr. Willard E. Bleick, Gibbs and Cox, Incorporated, New York, N. Y.;
Mr. Augusto Bobonis, University of Puerto Rico;
Professor Alexander William Boldyreff, University of Arizona;
Dr. Achille Capecelatro, New York, N. Y.;

Mr. George P. Cowan, St. John's University, Brooklyn, N. Y.;
Miss Mary Elizabeth Agnes Dever, Manhattanville College of the Sacred Heart;
Dean Alfonso Elder, North Carolina College for Negroes, Durham, N. C.;
Miss Frances E. Falvey, Ward-Belmont School, Nashville, Tenn.;
Professor Joseph Homer Forrest, Langston University, Langston, Okla.;
Mr. Franklin H. Fowler, Jr., New York, N. Y.;
Mr. Arthur Tapley French, State Teachers College, Winona, Minn.;
Mr. John Raymond Fritz, Wyomissing Polytechnic Institute, Wyomissing, Pa.;
Mr. Robert Elston Fullerton, Syracuse University;
Professor Andre Gleyzal, St. Michael's College, Winooski Park, Vt.;
Dr. Hans-Karl Hammer, Texas Company Research Laboratories, Beacon, N. Y.;
Mr. Wesley Turnell Hanson, Technical High School, Atlanta, Ga.;
Dr. George Edward Hay, Armour Institute of Technology;
Dr. Edward Helly, New York, N. Y.;
Dr. Morris Hendrickson, Ohio State University;
Mrs. Frances Louise Campbell Hinds, George Pepperdine College, Los Angeles, Calif.;
Sister Mary Charlotte Holland, St. Xavier College for Women, Chicago, Ill.;
Professor Elvin Albert Hoy, University of Hawaii;
Mr. Robert Sigurd Jacobson, Luther College;
Mr. E. A. Johnson, Phoenix, Ariz.;
Mr. Andrew Richard Kirby, College of the City of New York and Fordham University;
Mr. Calvin John Kirchen, Itasca Junior College, Coleraine, Minn.;
Mr. Stephen Anthony Kiss, Standard Oil Development Company, New York, N. Y.;
Mr. Richard Frederick Koch, Bard College, Columbia University;
Mr. Paul Herbert Kratz, Temple University;
Mr. Joseph S. Leech, College of Mines and Metallurgy, El Paso, Texas;
Mr. John N. B. Livingood, Gettysburg College;
Mr. Jack Lorell, Brown University;
Mr. Charles Edward McCauley, Woodstock College, Woodstock, Md.;
Mr. Charles B. McMullen, State Teachers College, Hyannis, Mass.;
Dr. Joanna Mayer, Xavier University, New Orleans, La.;
Mr. Herman Meyer, University of Chicago;
Professor José M. Mijares, Mapua Institute of Technology, Manila, P. I.;
Mr. Fred A. Miller, Wentworth Military Academy, Lexington, Mo.;
Dr. Richard von Mises, Harvard University;
Mr. Nathan Morrison, Division of Placement of Unemployment Insurance, New York State Department of Labor, New York, N. Y.;
Professor Mario John Nardelli, Fordham University;
Professor Edwin Burman Ogden, Union College, Lincoln, Neb.;
Mr. William Gray Palm, Jr., University of Nevada;
Mr. George William Patterson, National Bureau of Standards, Washington, D. C.;
Professor William Anderson Patterson, Fenn College, Cleveland, Ohio;
Professor Francisco Perez, University of the Philippines;
Miss Edris Pauline Rahn, Hayward Union High School, Hayward, Calif.;
Reverend John H. Raymond, St. Martin's College, Lacey, Wash.;
Dr. Paul V. Reichelderfer, Ohio State University;
Dr. Irving F. Ritter, New York University;
Mr. Walter C. Roberts, Glendale Junior College, Glendale, Calif.;
Professor Alexander Russ, Mississippi Woman's College, Hattiesburg, Miss.;

Professor Thomas Justin Saunders, University of San Francisco;
Reverend Maurice Andrew Scheier, St. Bonaventure College, St. Bonaventure, N. Y.;
Dr. Herman M. Schwartz, Duquesne University;
Dr. Andrew Sobczyk, Oregon State College;
Miss Ethel Sutherland, Teachers College, Columbia University;
Mr. Francis Xavier Sutton, Princeton University;
Professor Hubert M. Thaxton, Agricultural and Technical College of North Carolina, Greensboro, N. C.;
Dr. George Raymond Thurman, University of Missouri;
Professor Gerhard Tintner, Iowa State College;
Mr. Louis Bryant Tuckerman, III, Princeton University;
Professor Enrique T. Virata, University of the Philippines;
Dr. Abraham Wald, Columbia University;
Professor Marjorie Watson, Huntingdon College, Montgomery, Ala.;
Mr. William Wernick, New York, N. Y.;
Mr. Hugh S. Westbrook, U. S. Army Air Corps, Chicago, Ill.;
Mr. John Davis Williams, Princeton University.

It was reported that the following had been elected as nominees on the Institutional Memberships of the institutions indicated:

California Institute of Technology: Messrs. J. P. LaSalle, A. B. Mewborn, and J.R. Woolson.
Duke University: Messrs. Paul Civin and Milton Preston Jarnagin, Jr.
Pennsylvania State College: Messrs. Paul M. Kendig (Schuylkill Undergraduate Center, Pottsville, Pa.) and H. H. Reichard (Hazleton Undergraduate Center, Hazleton, Pa.).
Wayne University: Mr. Herbert Kapfel Brown.
Yale University: Messrs. Bengt Edvin Carlson, Eli Sherman Grable, and Wendell Gayton Swope.

It was announced that Dr. Joseph Gillis of Queen's University, Belfast, Northern Ireland, and Professor John Greenlees Semple of King's College, University of London, have been admitted to membership in the Society in accordance with the reciprocity agreement with the London Mathematical Society.

The Secretary announced that the meeting in Seattle, arranged to be held with the American Association for the Advancement of Science, is set for June 20, 1940. It was voted to hold an autumn meeting at Wayne University on November 22–23, 1940, and another at the University of California at Los Angeles on November 23; also a spring meeting at the University of Chicago on April 11–12, 1941. An invitation to hold the Annual Meeting of 1941 at the University of Florida was received.

The following appointments by President G. C. Evans to the four Committees on Invited Addresses were reported: for the Annual and Summer Meetings, Professor L. M. Graves (two years); for Eastern Meetings, Professors Einar Hille (two years) and J. L. Walsh (one

year to replace Dean L. P. Eisenhart, resigned); for Western Meetings, Professor E. W. Chittenden (two years); for Far Western Meetings, Professor Gabor Szegö (two years).

On recommendation of the Editorial Committee on Colloquium, a book by Professor Norman Levinson entitled *Tauberian Theorems and Gap Theorems* was accepted for publication in the Colloquium Series.

A special and unusual program is being arranged for the Summer Meeting of 1941 to be held at the University of Chicago in connection with the fiftieth anniversary of the founding of that institution. The following Committee on Arrangements was appointed: Professors L. M. Graves (chairman), A. A. Albert, W. L. Ayres, G. A. Bliss, W. D. Cairns, L. R. Ford, E. J. Moulton, and Dr. R. G. Sanger. A special Committee on Program was also appointed, consisting of Professors A. A. Albert (chairman), W. L. Ayres, G. A. Bliss, and T. H. Hildebrandt.

Titles and cross references to the abstracts of the papers read follow below. Papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 8 were read before the section for Analysis; those numbered 9 to 18 before the section for Topology; those numbered 19 to 24 before the section for Algebra; those numbered 25 to 30 before the section for Applied Mathematics, Geometry and Logic; those numbered 31 to 36 before the general session on Saturday morning; and papers numbered 37 to 92 were read by title. Paper 11 was read by Dr. Eilenberg, paper 29 by Professor Crout, and paper 33 by Dr. Schilling. Dr. Bergman was introduced by Professor W. T. Martin, Dr. Hua by Professor Hermann Weyl, Mr. Hurwitz by Professor G. D. Birkhoff, and Mr. Western by Dr. D. W. Hall.

1. Paul Civin: *Inequalities for trigonometric integrals.* Preliminary report. (Abstract 46-5-248.)

2. F. J. Weyl: *On the defect relation for meromorphic curves.* (Abstract 46-5-346.)

3. Stefan Bergman: *On the surface integrals of functions of two complex variables.* (Abstract 46-5-235.)

4. H. L. Krall: *Orthogonal polynomial solutions of a certain fourth order differential equation.* (Abstract 46-5-289.)

5. E. W. Titt: *On the characteristic theory of a first order partial differential equation.* (Abstract 46-7-367.)

6. R. P. Boas: *Univalent derivatives of entire functions.* (Abstract 46-5-238.)

7. Wilfred Kaplan: *Singularities of a curve-family on a surface, with applications to differential equations.* (Abstract 46-5-285.)

8. Michael Golomb: *On the boundary-value problems of the equation* $\Delta u = g(x, y)$ *in the infinite half-strip.* (Abstract 46-5-274.)

9. J. L. Kelley: *On the hyperspaces of a continuum.* (Abstract 46-5-287.)

10. P. M. Pepper: *Concerning pseudo-planar-quintuples.* (Abstract 46-5-313.)

11. Samuel Eilenberg and R. L. Wilder: *Uniform local contractibility.* (Abstract 46-5-269.)

12. A. D. Wallace: *On separation spaces.* (Abstract 46-7-368.)

13. G. E. Schweigert: *Equivalence of pointwise periodic and interior transformations on dendrites.* (Abstract 46-5-322.)

14. S. S. Cairns: *Triangulated manifolds which are not Brouwer manifolds.* (Abstract 46-7-355.)

15. O. G. Harrold (National Research Fellow): *Minimal coverings of Peano spaces by maps of a circle.* (Abstract 46-5-276.)

16. I. E. Segal: *Certain topological rings.* (Abstract 46-7-366.)

17. P. A. White: *On certain relatively non-alternating transformations.* (Abstract 46-5-347.)

18. D. W. Hall: *On arc and tree preserving transformations.* (Abstract 46-7-360.)

19. H. J. Riblet: *Factorization of differential ideals in an algebraic differential field.* (Abstract 46-5-318.)

20. H. S. M. Coxeter: *The binary polyhedral groups, and other generalizations of the quaternion group.* (Abstract 46-5-252.)

21. Olaf Helmer: *A new type of transcendence proof.* (Abstract 46-5-278.)

22. Richard Brauer: *On the Cartan invariants of a group of finite order.* (Abstract 46-5-239.)

23. J. E. Eaton: *On the extension of groups.* I. *Normal extensions.* II. *Non-normal extensions.* (Abstract 46-7-359.)

24. D. C. Murdoch: *A characterization of abelian quasi-groups.* (Abstract 46-5-304.)

25. M. H. Martin: *Quasi-Lagrangian systems.* Preliminary report. (Abstract 46-7-364.)

26. J. M. Clarkson: *An involutorial line transformation associated with a quadric.* (Abstract 46-5-250.)

27. Harry Bateman: *Hulthen's integral equation.* Preliminary report. (Abstract 46-5-231.)

28. H. B. Curry: *A formulation of recursive arithmetic.* (Abstract 46-5-253.)

29. F. B. Hildebrand and P. D. Crout: *A least square procedure for*

improving a method for solving integral equations by polynomial approximation. (Abstract 46-3-175.)

30. D. T. McClay: *On certain manifolds of somas.* (Abstract 46-5-292.)

31. J. H. Roberts: *A theorem on dimension.* (Abstract 46-5-319.)

32. J. L. Synge: *On the electromagnetic two-body problem.* (Abstract 46-5-329.)

33. Saunders MacLane and O. F. G. Schilling: *Normal algebraic number fields.* (Abstract 46-5-294.)

34. E. J. McShane: *On the second variation in certain anormal problems of the calculus of variations.* (Abstract 46-5-209.)

35. Oystein Ore: *The extension problem for groups.* (Abstract 46-5-311.)

36. A. D. Michal: *Higher order differentials of functions with arguments and values in topological abelian groups.* (Abstract 46-5-298.)

37. R. P. Agnew: *Some remarks on a paper entitled "General Tauberian theorems."* (Abstract 46-5-226-*t*.)

38. Stefan Bergman: *On a generalized Green's function and certain of its applications.* (Abstract 46-7-353-*t*.)

39. R. P. Boas: *Expansions of analytic functions.* (Abstract 46-5-237-*t*.)

40. Leonard Carlitz: *A set of polynomials.* (Abstract 46-5-243-*t*.)

41. Leonard Carlitz: *On certain exponential sums.* (Abstract 46-5-244-*t*.)

42. Randolph Church: *Numerical analysis of certain free distributive structures.* (Abstract 46-5-245-*t*.)

43. Nancy Cole: *The index theorem for a calculus of variations problem in which the integrand is discontinuous.* (Abstract 46-5-251-*t*.)

44. J. L. Coolidge: *Analytic systems of central conics in space.* (Abstract 48-7-356-*t*.)

45. Richard Courant: *On a generalized form of Plateau's problem.* (Abstract 46-7-357-*t*.)

46. M. M. Day: *A norm ergodic theorem.* (Abstract 46-5-257-*t*.)

47. M. M. Day: *Linear methods of summability.* II. (Abstract 46-5-258-*t*.)

48. A. González Domínguez: *Some theorems on the Hermite singular kernel.* (Abstract 46-7-358-*t*.)

49. J. J. DeCicco: *The affinelong near-Laguerre transformations.* (Abstract 46-5-259-*t*.)

50. E. L. Dodd: *The substitutive mean and certain subclasses of this general mean.* (Abstract 46-5-260-*t*.)

51. Jesse Douglas: *A new special form of the linear element of a surface.* (Abstract 46-5-262-*t*.)

52. Abe Gelbart: *On the growth of a function of two complex variables satisfying certain partial differential equations.* (Abstract 46-5-272-*t*.)

53. O. G. Harrold (National Research Fellow): *Characterizations of a class of continua by means of continuous functions.* (Abstract 46-5-275-*t*.)

54. M. H. Heins: *A note on a theorem of Radó.* (Abstract 46-5-207-*t*.)

55. L. K. Hua: *On Waring's problem for cubic polynomial summands.* (Abstract 46-5-280-*t*.)

56. L. K. Hua: *On the number of partitions of a number into unequal parts.* (Abstract 46-5-279-*t*.)

57. Henry Hurwitz: *On certain wave equations analogous to the Dirac equations.* (Abstract 46-5-281-*t*.)

58. Edward Kasner and J. J. DeCicco: *Families of curves conformally equivalent to circles.* (Abstract 46-5-286-*t*.)

59. B. O. Koopman: *Intuitive probability and sequences.* (Abstract 46-5-288-*t*.)

60. A. N. Lowan and Hyman Serbin: *On upper bounds for the remainder in the evaluation of differences.* (Abstract 46-7-363-*t*.)

61. A. N. Lowan and Jack Laderman: *On the distribution of errors in the computation of tables by differences.* (Abstract 46-7-362-*t*.)

62. E. J. McShane: *On the theory of relative extrema.* (Abstract 46-5-293-*t*.)

63. Margaret P. Martin: *A sequence of tests for the convergence and divergence of infinite series.* (Abstract 46-5-295-*t*.)

64. Philip Newman: *The geometry of the* (2, 2) *planar connex.* Preliminary report. (Abstract 46-5-306-*t*.)

65. Rufus Oldenburger: *Binary forms.* (Abstract 46-5-308-*t*.)

66. Oystein Ore: *Remarks on structures and group relations.* (Abstract 46-5-310-*t*.)

67. H. A. Rademacher and A. L. Whiteman: *On Dedekind sums.* (Abstract 46-5-317-*t*.)

68. Barkley Rosser: *An additional criterion for the first case of Fermat's last theorem.* (Abstract 46-5-320-*t*.)

69. Max Shiffman: *The Morse relations in the Plateau problem for several boundaries.* (Abstract 46-5-324-*t*.)

70. A. R. Schweitzer: *Concerning general abstract relational spaces.* (Abstract 46-3-190-*t*.)

71. D. C. Spencer: *On finitely mean valent functions.* (Abstract 46-5-326-*t*.)

72. D. C. Spencer: *On finitely mean valent functions.* II. (Abstract 46-5-327-*t*.)

73. Alvin Sugar: *On a result of Hua for cubic polynomials.* (Abstract 46-5-328-*t*.)

74. Olga Taussky and John Todd: *On determinants of quaternions.* (Abstract 46-5-330-*t*.)

75. W. J. Trjitzinsky: *Developments in the analytic theory of algebraic differential equations.* (Abstract 46-5-334-*t*.)

76. C. W. Vickery: *On cyclically invariant graduation.* (Abstract 46-5-336-*t*.)

77. T. L. Wade: *Subgeometries of projective geometries as theories of tensors.* (Abstract 46-5-338-*t*.)

78. H. S. Wall: *A class of power series bounded in the unit circle.* (Abstract 46-5-219-*t*.)

79. A. D. Wallace: *An analysis of non-alternating transformations.* (Abstract 46-5-339-*t*.)

80. A. D. Wallace: *On certain classes of subcontinua.* (Abstract 46-5-340-*t*.)

81. J. L. Walsh: *Note on overconvergent power series.* (Abstract 46-5-220-*t*.)

82. J. L. Walsh and W. E. Sewell: *On the degree of convergence of harmonic polynomials to harmonic functions.* (Abstract 46-5-341-*t*.)

83. J. V. Wehausen: *A space isomorphic to Hilbert space.* (Abstract 46-7-370-*t*.)

84. Louis Weisner: *Moduli of the roots of polynomials and power series.* (Abstract 46-5-343-*t*.)

85. Louis Weisner: *Power series, the roots of whose partial sums lie in a sector.* (Abstract 46-5-344-*t*.)

86. D. W. Western: *A method for analytic continuation.* (Abstract 46-5-345-*t*.)

87. Hassler Whitney: *On regular families of curves.* (Abstract 46-5-348-*t*.)

88. G. T. Whyburn: *Mapping theorems.* (Abstract 46-5-221-*t*.)

89. G. T. Whyburn: *On non-alternating transformations.* (Abstract 46-5-222-*t*.)

90. G. T. Whyburn: *On multicoherence.* (Abstract 46-7-371-*t*.)

91. G. T. Whyburn: *On singularities and inversion under interior transformations.* (Abstract 46-7-372-*t*.)

92. G. T. Whyburn: *Transformations on boundary curves.* (Abstract 46-7-373-*t*.)

T. R. HOLLCROFT,
Associate Secretary

BOOK REVIEWS

Orthogonal Polynomials. By Gabor Szegö. (American Mathematical Society Colloquium Publications, vol. 23.) New York, American Mathematical Society, 1939. 10+401 pp.

The general concept of orthogonal polynomials (OP) was introduced in Analysis in the fifties of the 19th century by Tchebyscheff, then professor at the University of St. Petersbourg. Naturally his pupils have carried on his research in this field. Thus in 1868 J. Sokhotsky in his thesis gives a general approach to OP, in some respects even more general than the one which prevails today. In 1884 Markoff's thesis again treats the general OP corresponding to absolutely continuous as well as discrete distribution-functions. In 1886 C. Possé published a monograph: *Sur quelques applications des fractions continues algébriques* where he considers the general OP related to

$$\int_a^b [f(y)/(x-y)]dy.$$

None of these works may be considered as a systematic treatment of the subject. In fact, time was not yet ripe for such treatment (it took almost a hundred years after the introduction of Legendre polynomials before a systematic treatment of these appeared—Heine's *Handbuch der Kugelfunktionen*). In 1932 N. Abramesco published an article in the Annales de Toulouse on the general OP. In 1934 the writer of these lines published a monograph in the *Mémorial des Sciences Mathématiques* designed to treat the theory of OP (applications are reserved for a second monograph in this collection) systematically and with as many details as limitation of space permitted.

The book under review gives in 400 pages a detailed and systematic treatment of the theory and applications of OP. It offers a very lucid and elegant exposition of the subject, to which Szegö himself made so many contributions. It is by no means a compilation of results already known. It presents much material which is new and important; many old results are presented in a novel setting: more precise or more general statements, new proofs. The author tried to bring the book as much up-to-date as possible and has generally succeeded.

Szegö's book consists of 16 chapters, the content of which may be summarized as follows. In Chapter I the author is marshaling the

necessary analytical tools: representation of nonnegative polynomials, bounds for the derivative of a polynomial or trigonometric sum, trigonometric and polynomial approximation of continuous functions, the concept of closure, linear functional operators, the Gamma-function, Bessel's functions. We also find here interesting "comparison theorems" of Sturm's type for linear differential equations. Chapter II treats the existence of OP and their representation in determinantal and multiple-integral form. The starting point here is the orthogonality property. The general principles are illustrated on the classical OP—of Jacobi, Hermite and Laguerre—also on OP corresponding to some special distribution-functions absolutely continuous and discrete. Chapter III deals with the general properties of OP: some extremal properties, closure, integral approximation in the sense of the least squares and p-powers, the all-important recurrence-relation, Darboux' formula. We also find here a discussion of the elementary properties of the zeros of OP and the corresponding mechanical quadrature formula. For the coefficients of the latter formula the author derives, in three different ways, the celebrated Tchebyscheff inequalities ("separation theorem"). The chapter closes with a brief discussion of algebraic continued fractions, in their relation to OP. Chapters IV, V, VIII, IX are devoted to the classical OP. In Chapters IV–V we find their differential equation, also representation by means of trigonometric and Bessel functions, hypergeometrical series, contour-integrals. In Chapter VIII the author makes use of the powerful tools of modern analysis in order to obtain asymptotic expressions for the classical OP in various parts of the interval of orthogonality (with an estimate of the remainder), also in the complex domain. This yields asymptotic estimates of their zeros. The interesting feature is the use of Bessel's functions which offers the advantage of extending the above asymptotic formulas to the endpoints of the orthogonality interval—of great importance in the study of expansion problems. Such expansions in series of the classical OP are taken up in Chapter IX: ordinary convergence and (C, k) summability for various values of k. These two chapters form, we believe, one of the most interesting parts of the book. Many results are new; many known results are presented in a decidedly more precise form. The study is based upon the corresponding Lebesgue constants. A masterly use of these constants enables the author to derive conditions for convergence and summability which, from the point of view of generality and precision, leave practically nothing to be desired. This is in sharp contrast to many older investigations, where the said conditions are frequently of an artificially restricted character. The

author is the more justified in devoting so much attention to the classical OP, for they form the backbone of the whole theory and serve as a yardstick in dealing with the general OP.

We turn back to Chapter VI. Here a study is made of the zeros of OP: analytic distribution, variation with a parameter. The general results are considerably elaborated for the classical OP (discriminant, extremal properties), where use is made of the corresponding differential equation (Sturm's methods). The results are extended to Fejér's generalization of Legendre polynomials, also to the generalized (non-orthogonal) Jacobi polynomials. In Chapter VII the author derives various inequalities for some special classes of OP (classical OP, monotonic weight-function) and related functions. These inequalities are of importance in mechanical quadratures, interpolation, also in evaluating certain integrals involving OP. We find here also a refinement, for polynomials, of the mean-value theorem of the integral calculus. Chapters X–XIII and XVI, based in the main upon the modern theory of functions of a complex variable, contains results due almost entirely to Szegö. Chapter X is devoted to the analytical representation of positive functions. This is made use of in the next chapter for a study of OP on the unit-circle (these were introduced in Analysis by Szegö). This discussion is continued and generalized in Chapters XII and XIII. It is extended in Chapter XVI to a rectifiable Jordan curve. The simple transformation $x=\frac{1}{2}(z+z^{-1})$ enables the author to return to the real domain and to derive important results concerning OP on a finite segment of the real axis. We note the asymptotic expression for a certain class of OP on $(-1, 1)$ corresponding to the so-called trigonometric weight-function (S. Bernstein). Szegö's method of deriving the said asymptotic expression is much simpler than that of Bernstein. Several general equiconvergence theorems for the real and complex domains constitute another important contribution. Chapters XIV and XV deal with interpolation and closely related mechanical quadrature (they are less extensive than other parts of the book). Here the author makes extensive use of the results developed in the preceding chapters. In Chapter XV a study is made of Lagrange and Hermite interpolation formulae based upon the zeros of OP: convergence, mean-convergence in the sense of pth powers. For the classical OP Szegö derives results more precise than those obtained by other writers. In Chapter XVI the author studies, mainly for the classical OP, the convergence of mechanical quadrature formulae based on Lagrange interpolation. Two types of mechanical quadrature formulae are considered: with and without weight-function. Concerning the latter type, the author takes up

the question of positiveness of the coefficients ("Cotes' numbers") which is of cardinal importance in this theory.

In closing the author gives a stimulating set of problems and exercises, also a considerable bibliography.

It goes without saying that the writer of a scientific book is the supreme judge in choosing its content. It is only natural for him to favor his own brain-children and, from amongst the works of others, those closely related to his own.

In the Introduction the author disclaims completeness of treatment. This disarming remark is necessary but not sufficient. In a book of this kind one would like to find as complete a treatment as possible, preparing the reader for future progress in various branches of the subject.

The omission of the problem of moments is, we believe, very regrettable. The problem of moments is important historically and is growing in importance both in pure and applied analysis. Its inclusion would have been much to the point. Moreover, it would have enabled the author (on the basis of the important results of M. Riesz) to give a complete treatment of closure for an infinite interval (the importance of closure in the theory of OP cannot be underestimated). As it is, the author discusses closure, in case of an infinite interval, for Laguerre and Hermite polynomials only. Furthermore, this inclusion of the moment-problem would have made possible a discussion of the limiting values of certain important definite integrals. Another regrettable feature is, we believe, the far too short space ($3\frac{1}{2}$ pages) devoted to continued fractions. The author remarks that continued fractions have been gradually abandoned as a starting point for the theory of OP. Be that as it may, one should not underrate continued fractions as a powerful tool in the theory and application of OP. We may mention a few other omissions: OP in several variables; other methods of summability of series of OP in addition to (C, k) summability, for example, the interesting results of Hille concerning Abel summability of expansions in series of Laguerre polynomials; application of OP to algebra (zeros of polynomials).

The reviewer did not pursue the thankless task of hunting misprints, minor errors or minor faults in exposition. While no book is free of these, the known thoroughness of the author offers sufficient assurance in this respect.

With all these omissions the beautifully printed book of Szegö is an excellent addition to the Colloquium Publications of the Society. It is a remarkable source of powerful methods and far-reaching re-

sults in an important field, which will inspire and stimulate further research.

J. SHOHAT

Structure of Algebras. By A. Adrian Albert. (American Mathematical Society Colloquium Publications, vol. 24.) New York, American Mathematical Society, 1939. 11+210 pp.

The study of algebras has been one of the most significant features of the present period in the history of mathematics; and in the theory of algebras a monument has been erected recording some of the characteristic traits of contemporary mathematical thought.

One may say that algebras have been pushed into the centre of attention by the publication of Dickson's *Algebras and their Arithmetics*; and from that moment on they have kept the interest of the mathematical public. In the meantime a great number of the problems has been solved, methods have been streamlined so that a moment propitious for the survey of the results has arrived. One of the principal actors in the movement has given an account of its results. The mathematical public certainly will be grateful for his effort, as he has been able to capture the inherent beauty of the theory of algebras and to communicate it to the reader.

The book may be divided roughly into two parts, the first being concerned with the general theory, the second containing applications to related problems. It should be mentioned at once that the theory of representations has been "put in its place"; that is, it appears as an application of the general theory and has not been used for the derivation of the results of the general theory.

The general theory of algebras may be defined as that part of the theory in which no special restrictions are imposed upon the field of reference. There are two main topics of discussion. The first is the reduction to simple algebras and the second the discussion of the simple algebras themselves.

The reduction theory proceeds in two steps. One shows first the existence of a radical and the semisimplicity of the algebra modulo its radical. This semisimple difference algebra may be represented by some subalgebra of the original algebra, provided the difference algebra stays semisimple under every scalar extension (this result belongs to a later phase of the theory, since changes of the field of reference have to be considered). But since both the investigation of nilpotent algebras and of the possible extensions of a nilpotent alge-

bra by a semisimple algebra have started only recently, the state of the theory allows one only to speak of a theory of semisimple algebras and not yet of a theory of algebras. The second step in the reduction theory is the proof of the theorems that every semisimple algebra possesses an identity element and is the direct sum of uniquely determined (not only in the sense of equivalence) simple algebras; and thus it is clear how to construct the most general semisimple algebra, as soon as the simple algebras are known. It should be noted that the theorems concerning semisimple algebras are gotten as special cases of corresponding theorems concerning algebras which may or may not possess a radical. However the reduction contained in these more general theorems does not lead to a class of algebras as tractable as the simple ones.

Every simple algebra is a direct product of a total matric algebra and a division algebra; and any two representations of this kind are conjugate, that is, may be transformed into each other by means of inner automorphisms. The inner automorphisms of simple algebras are exactly those automorphisms which leave the elements of the centrum invariant. The centrum of a simple algebra being a field, it may be taken as the field of reference, in which case we say that the the algebra is normal simple. Then the above theorem states that all the automorphisms of a normal simple algebra are inner. This theorem is supplemented by the fact that any equivalence of two simple subalgebras of a normal simple algebra may be induced by an automorphism of the containing algebra, provided the three algebras have the same identity element.

It follows from the above that the structure of a normal simple algebra is completely determined by two invariants: the one numerical, namely, the degree of the total matric algebra in question, the other a structural invariant, namely, the structure of the division algebra mentioned above. Thus one may be led to the following construction. One considers all the normal simple algebras over a fixed field F. Those normal simple algebras over F which are only distinguished by the nature of the total matric algebra involved and lead to the same division algebra are put into one class and these classes form an abelian group under direct multiplication (of its elements).

If the field N is a finite, normal, and separable extension of the field F, then N determines a subgroup of the group of classes of normal simple algebras over F as follows. There exist some normal simple algebras over F which contain N as a maximal subfield. The product of such an algebra by N is a total matric algebra over N, that is, is split by N; and each such algebra is a crossed product of N by the

Galois group of N over F. The classes of normal simple algebras over F which are split by N form a subgroup of the group of classes of algebras over F; and this subgroup is practically the same as the group of classes of associated factor sets of N over F. As each normal simple algebra over F is split by some such N, it suffices to consider these subgroups of the group of classes; and thus it is possible to use the theory of factor sets for proving theorems like the following one: every element of the group of classes is of finite order in this group; this order divides the degree of the characteristic division algebra and is divisible by every prime divisor of this degree.

As to the methods of proof employed in the derivation of this theory two tools ought to be mentioned which the author uses with much dexterity and success. There is first the commutator of a subalgebra in a containing algebra, that is, the set of all those elements in the containing algebra which permute with every element in the given subalgebra—the adoption of this term commutator may be regretted, as this concept is called centralizer everywhere outside this theory. The importance of the commutator may be seen from the following theorems. If some algebra is the direct product of two algebras B and C such that B is normal simple and such that B and C have the same identity, then C is the commutator of B in their direct product, that is, one factor of the direct product determines the other one in a unique fashion. More generally: the simple subalgebras of a normal simple algebra occur in pairs such that each member of such a pair is the commutator of the other one; and forming the commutator is an involutorial operation as far as simple subalgebras of a normal simple algebra are concerned.

Observing that multiplication of the elements of an algebra by a fixed element is a linear transformation of the algebra, that the linear transformations form a total matric algebra, that the transformations gotten by left-multiplication with an element form exactly the commutator of those gotten by right-multiplication (and conversely), and that the second of these sets of transformations is isomorphic, the first anti-isomorphic to the given algebra, one proves the theorem that the direct product of a normal simple algebra by its reciprocal (anti-isomorphic) is a total matric algebra. This theorem is so powerful a weapon in the theory because it permits a reduction of the proofs of theorems concerning normal simple algebras to the proofs of the corresponding theorems concerning total matric algebras.

The following partial converse of the last theorem ought to be mentioned. The direct product of two division algebras is a total matric algebra if, and only if, these two division algebras are reciprocal.

As an illustration of the author's methods in handling these tools we indicate a proof of the theorem that every automorphism of a normal simple algebra is an inner automorphism (our reason for choosing just this example is that the author does not use his methods for a proof of this theorem and that the proof given in the text is inconclusive). If A is a normal simple algebra, A' its reciprocal, then their direct product is a total matric algebra $T=A\times A'$. If f is an automorphism of A, then there exists one and only one automorphism of T which induces f in A and leaves every element in A' invariant. As every automorphism of a total matric algebra is an inner automorphism, there exists some element t in T such that transformation by t induces in T just this automorphism. This element belongs clearly to the commutator of A' in T. Since the commutator of A' in T is just A, it follows that t belongs to A so that f is the inner automorphism of A, induced by this element t.

As a first application of the theory of algebras, the theory of representations has been treated. This theory is no longer concerned with the study of the structure of algebras but with the study of the structure of homomorphisms between algebras. If both algebras are considered over the same field, then there exists however a connection between the two theories which is very near to an equivalence. The following theorems may be mentioned as typical examples of this situation. A representation of the algebra A over F in the subalgebra A^* of a total matric algebra M over F is irreducible if, and only if, A^* is simple and the commutator of A^* in M is a division algebra; it is fully decomposable if, and only if, A^* is semisimple; it stays irreducible under every scalar extension if, and only if, A^* itself is a total matric algebra. On the other hand it must be admitted that by its very definition the theory of representations is much more complicated than the theory of algebras proper; and thus both theories are greatly improved by doing the simpler thing first and using it then for the derivation of the more involved theory.

The most important application of the general theory and perhaps the most interesting part of the whole work is the enumeration of the normal simple algebras over finite algebraic number fields or—what amounts to the same thing—of the simple algebras over the field of rational numbers. The problem of how to construct all these algebras is settled by the famous theorem that every simple algebra over the field of rational numbers is not only similar to, but may actually be represented as, the crossed product of a cyclic extension of an algebraic number field by its cyclic Galois group. In order to classify these algebras the author proceeds as follows. If the field F is a finite ex-

tension of the field of rational numbers, V a valuation of F and A a normal simple algebra over F, then denote by F_V the essentially uniquely determined derived field of F with respect to the valuation V and by A_V the direct product of A and F_V (that is, the scalar extension of A by F_V). A reduction of the problem "in the large" to a problem "in the small" is contained in the theorem that the normal simple algebras A and B over F are equivalent if, and only if, A_V and B_V are equivalent for every valuation V. This theorem is a comparatively simple consequence of the following lemma (which the author states without giving a proof): an element in F is the norm of an element in the given finite and normal extension N of prime degree over F if, and only if, it is a norm of an element in N_V for every valuation V of N. Thus it suffices to determine these algebras A_V. There exists at most a finite number of valuations V of F such that A_V is not a total matric algebra. If A_V is not a total matric algebra, and if V is an archimedian valuation, then A_V is a real quaternion algebra. If A_V is not a total matric algebra, and if V is a non-archimedian valuation, then A_V is completely determined by two numerical invariants one of which is just the degree of the division algebra characteristic for the class of A_V whereas the other one is derived from a certain normal form of representing A_V as a crossed product.

There is a great number of further equally interesting applications of the theory which find treatment in this book. But it would lead too far to discuss them here at great length. Suffice it to mention such topics as the cyclic systems, the modern theory of Riemann matrices and of involutions, and more special problems like the enumeration of all normal division algebras of degrees three and four.

The standard source of reference for the theory of algebras has been in recent years Deuring's report on this topic. As to the material covered none of these works has been a subset of the other and even in their treatment of the common parts they differ widely. In the reviewer's opinion Albert's text is the more easily accessible one of the two, so much so that he feels that the book will serve admirably as an introduction into the theory to all those who know the rudiments of present-day algebra. The expert on the other hand will find this exposition interesting, stimulating and useful both because of the new material included and because of the more "streamlined" treatment of it.

An extensive bibliography of recent publications on related topics has been added which will be welcome and helpful to every student of the field.

REINHOLD BAER

The Classical Groups. By Hermann Weyl. Princeton, University Press, 1939. 11+302 pp.

It is a curious fact that while almost all the textbooks on higher algebra written prior to 1930 devote considerable space to the subject of invariants, the recent ones written from the axiomatic point of view disregard it completely. Because of this neglect the phrase "invariant theory" is apt to suggest a subject that was once of great interest but one that has little bearing on modern algebraic developments. For this reason it is an important and original accomplishment that Professor Weyl has made here in connecting the theory of invariants with the main stream of algebra and in indicating that the subject has a future as well as a distinguished past.

In his treatment the theory of invariants becomes a part of the theory of representations. The most natural way to begin the study of invariants of a particular group is therefore to determine its representations. A large part of the book is concerned with this problem as it applies to the "classical" groups $GL(n)$, the full linear group; $O(n)$, the orthogonal, and $S(n)$, the symplectic (complex or abelian) group.

In discussing the representations of an abstract group $\mathfrak{g}$ one finds it convenient to adjoin to the set of representing matrices their linear combinations. The resulting set is an algebra, the enveloping algebra of the original set, and defines a representation of a certain abstract algebra, the group algebra, that is completely determined by $\mathfrak{g}$ and the field of the coefficients. In this way the theory of algebra is applicable. The author has gone somewhat beyond his immediate needs in discussing this domain. Consequently his book may serve also as an excellent introduction to this theory.

The book begins with a brief review of the basic concepts of field, abstract group, vector space, linear transformation (matrix) and representation of a group. These are used to give the following definition of an invariant: Let $x, y, \cdots$ be variable vectors varying in different representation spaces of an abstract group $\mathfrak{g}$ and let $A(s), B(s), \cdots$ be the corresponding matrices. A polynomial $f(x, y, \cdots)$ in the coordinates of $x, y, \cdots$ is an invariant of $\mathfrak{g}$ if $f(A(s)x, B(s)y, \cdots) = f(x, y, \cdots)$ for all s in $\mathfrak{g}$. Relative invariants and covariants are defined along similar lines. The central problems to which this definition gives rise are: (1) Given a class of invariants, to determine, if possible, a basis, that is, a finite number $I_1, I_2, \cdots, I_r$ of these invariants such that every invariant in the class is expressible as a polynomial in the I's. (2) To determine a fundamental set of relations obtaining between the basic invariants. The solution of these prob-

lems for vector invariants[1] of the unimodular group $SL(n)$ and of the orthogonal group is the subject of Chapter II. There are two essential steps in the proofs, the formal part by which the theorems are reduced by certain identities (Capelli's) to the cases where the number of vector variables of each kind does not exceed $(n-1)$ and the numerical argument which disposes of the reduced cases. The proofs are less direct for $O(n)$ than for $SL(n)$. For here it is necessary to suppose first that the underlying field is real and then to extend the results to arbitrary fields of characteristic 0 by using Cayley's parametrization of the "general" orthogonal matrix of determinant 1 as $(E-S)(E+S)^{-1}$ where E is the identity and S is an arbitrary skew-symmetric matrix.

The third chapter introduces the theory of algebras. The treatment given is based on the concept of a completely reducible matrix algebra. It is shown that any representation of an algebra $\mathfrak{A}$ of this type is completely reducible and there is a reciprocity between $\mathfrak{A}$ and the algebra $\mathfrak{B}$ consisting of the matrices commutative with those of $\mathfrak{A}$: $\mathfrak{B}$ is completely reducible and $\mathfrak{A}$ in turn may be characterized as the set of matrices commutative with those of $\mathfrak{B}$. Together with Schur's lemma this result yields Wedderburn's fundamental theorems. The methods used here are well suited for handling problems of the type discussed in this book, where algebras occur in concrete form as sets of matrices. More abstract methods would be required if one wished to obtain these theorems in their most general form. In the second half of the chapter an independent treatment of group algebras is given. Their complete reducibility and reciprocity with the commutator algebras is derived in a very elementary way. As a consequence the results, particularly the latter, are given a much more explicit form than in the general situation.

After this machinery has been set up, it is possible to determine the (integral) representations of $GL(n)$. One must establish first the complete reducibility of the tensor representation $A \to A \times A \times \cdots \times A$ (Kronecker product of f factors) and to split this representation into its irreducible parts. This is accomplished by showing that the enveloping algebra $\mathfrak{A}_f$ is the commutator of a certain representation of σ_f, the group algebra of the symmetric group on f letters. Explicit formulas for the idempotent elements of σ_f (Young symmetrizers)

[1] That is, functions of an arbitrary number of covariant and contravariant vectors where by a covariant vector we mean one belonging to the original representation $x \to Ax$ and contravariant refers to vectors in $\xi \to (A')^{-1}\xi$, A' the transposed matrix. There is no distinction between covariant and contravariant vectors if A is orthogonal.

yield the irreducible subspaces of tensor space. These results may be extended to the representation obtained by stringing along the diagonal the tensor representations of ranks 1, 2, $\cdots$, f. The enveloping algebra is here denoted as $\mathfrak{A}^{(O)}$. Now, if $A \to T(A)$ is any representation of $GL(n)$ whose coordinates are polynomials of degree not greater than f in the coordinates of A, then the enveloping algebra of $T(A)$ represents $\mathfrak{A}^{(O)}$. The general theory shows that $T(A)$ is completely reducible into parts similar to some of those contained in the tensor representations.

To obtain these results for $O(n)$ the author follows a procedure due to Brauer (used first by Weyl for $S(n)$). The complete reducibility of the tensor representation is established for the case of real fields. Its commutator algebra is determined by using the first main theorem for orthogonal invariants. Finally one obtains the enveloping algebra of the tensor representation by the reciprocity theorem. The tool by which the reality restriction is removed is again Cayley's parametrization. The real significance of the parametrization is brought out in the following beautiful theorem: The ideal generated by the relations defining an orthogonal matrix of determinant 1 is prime and has the generic zero $(E-S)(E+S)^{-1}$. To obtain the irreducible parts of tensor space relative to $O(n)$ a process of contraction is used. By this means the problem is reduced to one of decomposing certain spaces relative to the symmetric group. These results are then extended to the proper orthogonal group. The discussion of the symplectic group is reserved for a separate chapter. It parallels that of $O(n)$ and enables the reader to get a bird's-eye view of the entire structure as it appears up to this point.

In cases such as those under discussion here, in which the representations decompose into absolutely irreducible parts, a knowledge of these parts, though not their explicit construction, may be obtained by expressing the character of the given representation as a sum of primitive (irreducible) characters. This is done in Chapter VII for $GL(n)$, $O(n)$ and $S(n)$. The method is that of Weyl's "unitarian trick." One replaces these groups by certain compact Lie groups (thus, in place of $GL(n)$ one uses the group of unitary matrices). The characters of the latter may be obtained by using an invariant integration defined on the group manifolds. Since any algebraic relation with rational coefficients in the elements of the compact group $\mathfrak{u}$ holds also for the original group $\mathfrak{g}$, the formulas obtained for $\mathfrak{u}$ are valid for $\mathfrak{g}$ after being expressed as polynomials in the coefficients of the matrices. It is shown incidentally that the representations, determined by the earlier constructions form a complete set of continuous

irreducible representations for the compact groups. As an illustration of the use of characters, the author determines the irreducible components of the Kronecker product of irreducible representations.

In the first part of Chapter VIII the general definition of an invariant is specialized to the classical case. Here the representation space is determined by the coefficients of a homogeneous polynomial. The symbolic method as well as certain irrational methods are discussed as tools to obtain the first fundamental theorem. Hilbert's basis theorem for polynomial ideals settles the question of the finiteness of relations among the basic invariants. The symbolic method is then used to extend these results to general orthogonal and symplectic invariants. The ideas in the last half of the chapter cluster around the concept of a Lie group and its Lie algebra. The local properties of these groups are all expressible in terms of the algebras and if the group is simply connected one may readily pass to its properties in the large. On the other hand Lie algebras may be studied independently of the group theory and questions analogous to those regarding representations, invariants, and so on, may be raised. For compact Lie groups the powerful integration method finds another application here in a very simple derivation of the first fundamental theorem for general invariants. The last chapter resumes the discussion of algebras. A number of interesting results concerning automorphisms and direct products are obtained by matrix methods.

We have confined our description to the broad outlines of the subject. Space does not permit a detailed account of the interesting bypaths indicated by the author. We may mention, for example, a new formulation of Klein's Erlanger program, the close connection between representation theory and the theory of almost periodic functions and Fourier series, the topology of the classical groups. There are ample indications in the notes and bibliography to enable the reader to pursue these questions further.

In short, the book is heartily recommended to the present generation of algebraists as an introduction to a rich and rather neglected field and to those educated in the classical tradition for an insight into important recent algebraic ideas and their applicability to familiar problems.

N. JACOBSON

Aspects of the Calculus of Variations. Notes by J. W. Green after lectures by Hans Lewy. Berkeley, University of California Press, 1939. 6+96 pp.

As indicated in the introduction, the goal of the lecturer was to

acquaint the reader with a skeleton of methods such as he is apt to encounter in the calculus of variations, rather than to formulate results as generally as possible. The first chapter is devoted to a discussion of various problems, and the derivation of the first order necessary conditions for an extremum. Chapter II treats quadratic problems; in particular, the theory of integral equations with real symmetric kernel and boundary value problems associated with a second order linear differential equation. Chapter III, which deals with sufficient conditions, is extremely brief; nowhere in the lectures is the Weierstrass $\mathcal{E}$-function mentioned. In Chapter IV the work of Lewy (Mathematische Annalen, vol. 98 (1928), pp. 107–124) on the absolute minimum for nonparametric problems in the plane is presented. Chapter V is concerned with harmonic functions and associated boundary value problems; this chapter is preliminary to the discussion of the problem of Plateau and conformal mapping given in Chapter VI. In this last chapter the problem of Plateau is solved for a single contour in three dimensions; the method of proof is that of Courant (Annals of Mathematics, (2), vol. 38 (1937), pp. 676–724).

W. T. REID

Die Zylinderfunktionen und ihre Anwendungen. By R. Weyrich. Leipzig, Teubner, 1937. 5+137 pp.

A cylinder function may be defined as any solution of Bessel's differential equation. They include functions of the first kind, $J_p(z)$, also called Bessel functions, which are regular at $z=0$, and functions of the second kind which are not regular at $z=0$. There has been considerable confusion in the notation and canonical form of the cylinder functions of the second kind. They have been denoted by Y, G, K, and so on, by various authors and often the same notation is used with different meanings. Following the tables of Jahnke and Emde, our author uses the notation $N_p(z)$ for functions of the second kind and calls them Neumann functions. They are the same as those denoted by $Y_p(z)$ by Nielsen and also by Watson. (See Watson, *Theory of Bessel functions*, p. 57, for a further discussion.) In addition there are the Hankel cylinder functions, which are really functions of the second kind, defined by

$$H_p^{(m)}(z) = J_p(z)+i(-1)^{m+1}N_p(z), \qquad m=1, 2, \ i=(-1)^{1/2}.$$

The importance of the Hankel functions arises from the fact that alone among the cylinder functions $H_p^{(m)}(re^{i\theta})\to 0$ as $r\to\infty$, provided that $m=1$, $0\leqq\theta\leqq\pi$, and that $m=2$, $\pi\leqq\theta\leqq 2\pi$.

One way to introduce cylinder functions is to define them, as above,

as solutions of Bessel's differential equation. Our author, however, follows a different path. His starting point is the wave equation, from which he derives first plane waves, and then, by superposition of these, spherical and cylindrical waves. This leads immediately to the Sommerfeld integral representation of the Hankel functions, which then serve as the basis for defining the J and N functions and for deriving the important properties of the cylinder functions.

The author discusses the following aspects of cylinder functions: power series, asymptotic expansions of Hankel and Debye, various integral representations, recurrence relations, zeros, definite and indefinite integrals, boundary value problems and applications. It will be noted that the author has succeeded in covering the most important topics in a remarkably small number of pages. But the value of the book must not be judged by its brevity. It contains a carefully planned exposition of the theory and will serve as a valuable study and reference book.

C. A. SHOOK

Differentialgeometrie der Kurven und Flächen und Tensorrechnung. By Václav Hlavatý. Groningen, Noordhoff, 1939. 11+569 pp.

This treatise presents a large portion of the classical differential geometry of one and two dimensional subspaces of ordinary euclidean space. Just enough vector and tensor analysis is given to enable the reader to manage profitably the abbreviated symbolism. Definitions and results are stated in such a way as to generalize readily to higher dimensions.

The first chapter is devoted to curves. After the theory is developed in terms of a general parameter, an account is given of the various specializations arising from the use of the arc length as parameter. In particular, the construction of a curve from its curvature and torsion is treated carefully.

The second chapter concerns those properties of a surface which depend only on its metric tensor. The absolute differential is used systematically. There is an unusually full discussion of the applicability of surfaces, including explicit equations for developing a surface on a plane or on a surface of revolution.

The normal to a surface leads, on differentiation, to a tensor associated with the behavior of the surface toward the ambient space. In the third chapter, those properties are discussed which depend on this (second fundamental) tensor. A feature of this chapter is the discussion of the explicit construction of surfaces having prescribed first or first and second fundamental tensors.

The fourth chapter contains applications to a variety of special surfaces, notably ruled surfaces and minimal surfaces. There is no treatment of quadrics nor of congruences of lines.

The book is carefully written, and a high level of explicitness is maintained in the details of the argument and in the treatment of special cases. The reviewer feels that two exceptions to this statement may confuse readers unfamiliar with tensor analysis. "Tensor" is so defined that there is no distinction between a tensor and the set of its components in any particular coordinate system. This distinction—analogous to that between the number two and a couple of apples—is the key to the geometric significance of tensors, and this point seems to have been obscured by the form given to the definition.

In the second place, "differential invariant" is defined in the algebraic sense—as a function which is transformed by substituting for the old variables the appropriate functions of the new variables and multiplying by a certain power of the Jacobian. It is then stated that "The problem of metric differential geometry . . . consists in the study of the invariants . . . which can be obtained from the equations of the surface (or curve)." As examples, two invariants for each dimension are constructed, the whole treatment covering about five pages. No further mention is made of the concept. This procedure appears open to several objections. First, no explicit connection is established between the remainder of the text and "the problem of differential geometry." Second, it seems pedagogically unsound to introduce such a basic concept without giving the reader more definite occasion to become familiar with its applications and to grasp at least one precise significance of a word having a confusing variety of meanings. Finally, the term has been so used in other books that tensors are "invariants." If a practice opposed to this usage is adopted, a word of caution to readers seems desirable.

The book has an ingenious index which greatly enhances its usefulness as a source for reference. There is no bibliography, nor exact reference to sources, nor exercises. In dealing with surfaces, roman numerals I and II are used as indices for quantities pertaining to the surface; the gain in emphasis seems offset by the visual hesitation in distinguishing I II from II I.

Chiefly for its thoroughgoing use of tensor methods, the book is a valuable complement to the available treatises. It is distinguished from them by numerous interesting details in the presentation.

F. A. FICKEN

NOTES

Lattice Theory, by Assistant Professor Garrett Birkhoff of Harvard University, has just been published as volume 25 of the series of American Mathematical Society Colloquium Publications. The volume contains 155 pages. The list price is $2.50, with a discount of twenty-five per cent to members. Orders should be sent to the Society office at 531 West 116th Street, New York City.

In March of this year the Mathematical Society of Hamburg celebrated the two hundred fiftieth anniversary of its founding.

The National Council of Teachers of Mathematics will hold its 1940 summer meeting July 1-3 in Milwaukee, Wisconsin. The theme of the meetings is *How mathematics serves our community.*

The fourth annual mathematical symposium at the University of Notre Dame was held April 10–11, 1940, on the foundations of topology. The following papers were presented: *On contiguous points*, Professor R. L. Moore; *The topology of lumps*, Professor Karl Menger; *Classification of topological functions*, Professor E. W. Chittenden; *The equal generality of "convergence," "closure," and "neighborhoods,"* Dr. J. W. Tukey; *Partially ordered sets and topology*, Dr. A. N. Milgram; *The foundations of algebraic topology*, Professor Solomon Lefschetz; *Lattices and connectivity*, Dr. Henry Wallman.

At the meeting of the National Academy of Sciences in Washington, D. C., April 22-23, papers were presented by Professors Harry Bateman, G. D. Birkhoff, Gregory Breit, Edward Kasner, and Marston Morse.

Professor W. D. Cairns, Secretary-Treasurer of the Mathematical Association of America, has announced that the Department of Mathematics of the University of Toronto has won the first prize of $500 in the third annual William Lowell Putnam Mathematical Competition, held March 2, 1940. The members of the team were W. J. R. Crosby, J. C. Maynard, G. H. K. Strathy. The second prize of $300 is awarded to the Department of Mathematics of Yale University, members of whose team were J. E. Brewster, A. M. Gleason, G. R. MacLane. For the third prize of $200 there is a tie between the Department of Mathematics of Columbia University and the Department of Mathematics of Cooper Union Institute of Technology. The members of the Columbia University team were Laurence Annenberg, Julius Ashkin, Paul Marcus; the members of the Cooper

Union team were Murray Klamkin, Benjamin Lax, Samuel Manson. In addition to these prizes to the departments of mathematics with winning teams, a prize of $50 each is awarded to the following five persons whose scores ranked highest in the six-hour examination (names arranged alphabetically): W. J. R. Crosby, University of Toronto; A. N. Gleason, Yale University; E. L. Kaplan, Carnegie Institute of Technology; J. C. Maynard, University of Toronto; R. M. Snow, George Washington University. One of these five will be chosen to receive a $1000 year scholarship at Harvard University, this award to be announced later. The members of the four winning teams will receive individual cash awards according to the ranks of their teams, and all individuals receiving awards will also receive medals. Honorable mention has been awarded this year to three teams and to five individuals. The teams are from the Department of Mathematics, University of California at Berkeley, members of the team being Julia H. Bowman, W. M. Kincaid, C. W. Lippman; the Department of Mathematics, University of California at Los Angeles, members being Richard Arens, Robert James, Harold Shniad; and the Department of Mathematics, Carnegie Institute of Technology, members being S. N. Foner, E. L. Kaplan, W. E. Stuermann. The five individuals receiving honorable mention are: G. R. MacLane, Yale University; Samuel Manson, Cooper Union Institute of Technology; Paul Marcus, Columbia University; G. H. K. Strathy, University of Toronto; J. E. Wilkins, Jr., University of Chicago.

Among those recently elected to fellowships in the Royal Society of Edinburgh are Dr. A. F. Buchan of James Gillespie's School, Edinburgh, Dr. H. Davenport of the University of Manchester, and Professor H. S. W. Massey of University College, London.

The following persons have been awarded fellowships by the Guggenheim Foundation: Dr. Jesse Douglas, Brooklyn, N. Y., Professor R. L. Wilder of the University of Michigan, Associate Professor A. F. Wintner of the Johns Hopkins University, and Assistant Professor Gordon Pall of McGill University.

Dr. A. S. Gale of the University of Rochester has been made dean emeritus of the physical sciences.

Professor Hermann Weyl of the Institute for Advanced Study has been elected a member of the National Academy of Sciences.

Professor G. Vranceanu of the University of Cernauti has been appointed to a professorship at the University of Bucharest.

Assistant Professor J. E. Eaton of Hofstra College is now head of the department of mathematics.

Dr. K. O. Friedrichs of New York University has been promoted to an associate professorship.

Associate Professor Rufus Oldenburger of Armour Institute of Technology will deliver a series of lectures on modern algebra and dynamics at the University of Mexico during July and August.

Assistant Professor Mina S. Rees of Hunter College has been promoted to an associate professorship.

Dr. M. S. Robertson of Rutgers University has been promoted to an assistant professorship.

Associate Professor D. J. Struik of Massachusetts Institute of Technology has been promoted to a professorship.

Professor L. C. Karpinski of the University of Michigan will be on leave for the first semester of 1940–1941.

Professor A. Zygmund, formerly of the University of Wilno, has accepted a position at Mt. Holyoke College for 1940–1941.

Sir Thomas Heath died on March 16, 1940, at the age of seventy-eight years.

The Polish Government Information Department has reported the death of Professor A. Hoborski.

Professor J. H. Michell of the University of Melbourne died on February 3, 1940, at the age of seventy-six years.

The death of Professor Ernesto Pascal of the University of Naples on January 25, 1940, has been reported.

Professor Otto Toeplitz, formerly of the University at Bonn, died on March 15, 1940.

Colonel C. P. Echols, retired professor of mathematics of the United States Military Academy at West Point, died May 21, 1940. He had been a member of the Society for many years.

Associate Professor Beulah Russell of the College of William and Mary died February 23, 1940.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

353. Stefan Bergman: *On a generalized Green's function and certain of its applications.*

A theorem of Blaschke states: $\sum_{\nu=1}^{\infty} \lg |a_\nu| > -\infty$, $|a_\nu| < 1$, is the necessary and sufficient condition for the existence of a nonnegative harmonic function $H(z)$, $H(0) < \infty$, $z \in \mathfrak{E}^2 - \mathfrak{S}_\nu\{a_\nu\}$, $\mathfrak{E}^2 = E[|z| < 1]$ which possesses the property that $H(z) + \log|z - a_\nu|$, $\nu = 1, 2, \cdots$, is regular at $z = a_\nu$. Let $\mathfrak{M}^4 = E[|z_1| < 1, z_1 \in \mathfrak{B}^2(z_2)]$, $\mathfrak{B}^2(z_2) = E[z_1 = sh(z_2, \lambda)$, $0 \leqq s < 1$, $0 \leqq \lambda \leqq 2\pi]$, $h(z_2, \lambda)$, $h(z_2, 0) = h(z_2, 2\pi)$, being for every λ an analytic function of z_2, $z_2 \in \mathfrak{E}^2$. In the paper (Compositio Mathematica, vol. 6 (1939), pp. 307–335) there was introduced for every domain $\mathfrak{M}^4$ an "extended class of functions" possessing properties analogous to those of harmonic functions. If $g(z_1, z_2)$ is in $\mathfrak{M}^4$ an analytic function, it is then possible to construct the "Green's function of the extended class" $\Gamma(z_1, z_2; g; \mathfrak{M}^4)$ vanishing on the boundary surface $E[|z_2| = 1, z_1 = h(z_2, \lambda), 0 \leqq \lambda \leqq 2\pi]$ such that $\Gamma(z_1, z_2; g; \mathfrak{M}^4) + \lg|g(z_1, z_2)|$ is a function of extended class. Let $g_\nu(z_1, z_2)$ designate a set of functions analytic in $\mathfrak{M}^4$ satisfying a certain additional restriction concerning the distribution of zero surfaces. The condition $\sum_{\nu=1}^{\infty} \Gamma(0, 0; g_\nu; \mathfrak{M}^4) < \infty$ is necessary and sufficient for the existence of a nonnegative function $H(z_1, z_2)$, $H(z_1, z_2) < \infty$ for $(z_1, z_2) \in \mathfrak{M}^4 - \mathfrak{S}_\nu \mathfrak{G}_\nu^2$, $\mathfrak{G}_\nu^2 = E[g_\nu(z_1, z_2) = 0, (z_1, z_2) \in \mathfrak{M}^4]$ such that for every n, $H(z_1, z_2) + \sum_{\nu=1}^{n} \lg|g_\nu(z_1, z_2)|$, belongs to the extended class of functions of $\mathfrak{M}_{n_0}^4$, $n_0 \leqq n$, where $\lim_{n\to\infty} \mathfrak{M}_n^4 = \mathfrak{M}^4$. (Received March 28, 1940.)

354. L. M. Blumenthal: *A new concept in distance geometry, with applications to spherical subsets.*

If $\sigma \subset S$, a semimetric space, then S has σ-relative congruence indices $\{n, k\}$ with respect to a given class (Σ) of semimetric spaces provided any space Σ of (Σ) with more than $n+k$ pairwise distinct points is congruently contained in S whenever each n of its points is congruently contained in σ. This concept gives rise to new problems in distance geometry. For $\sigma = S$ the indices $\{n, k\}$ are called merely congruence indices of S with respect to (Σ). The notions of congruence and quasi congruence orders introduced by Menger correspond to congruence indices $\{n, 0\}$, $\{n, 1\}$, respectively, while $\{n, 2\}$ signifies a property weaker than quasi congruence order n but stronger than quasi congruence order $n+1$. Let $\kappa_{2,\rho}$ denote a cap with spherical radius ρ of the sphere $S_{2,r}$. It is shown that (1) if $\sigma = \kappa_{2,\pi r/2}$, with base circle removed, then $S_{2,r}$ has σ-relative congruence indices $\{4, 2\}$ with respect to all semimetric spaces (Σ); (2) the closed cap $\kappa_{2,\rho}$, $\rho < \pi r/2$, has congruence indices $\{4, 2\}$ with respect to (Σ); (3) $\kappa_{2,\rho}$, $\rho < r \cos^{-1} 1/3$, has indices $\{4, 1\}$. Analogous theorems for caps of $S_{n,r}$ are obtained. (Received April 12, 1940.)

355. S. S. Cairns: *Triangulated manifolds which are not Brouwer manifolds.*

This paper relates to problems involving Brouwer manifolds, differentiable manifolds, and polyhedral representations of complexes. A *Brouwer m-manifold* (L. E. J. Brouwer, Mathematische Annalen, vol. 71 (1912), pp. 97–115) is a triangulated topological m-manifold such that each star has a piecewise linear homeomorphic image in an euclidean m-space E^m. (I) A triangulated topological m-manifold is not necessarily a Brouwer manifold if $m>3$. This is one consequence of a constructed triangulation of an m-simplex ($m=3, 4, 5, \cdots$) not homeomorphic to any rectilinear triangulation of it. Other consequences follow. (II) Triangulated m-spheres exist ($m=3, 4, 5, \cdots$) which cannot be represented as convex polyhedral spheres in an E^{m+1}. (III) The lowest-dimensional euclidean space in which a complex K has a nonsingular polyhedral representation is not, for every K, invariant under subdivision. (IV) For every $m>3$, there exist stars of simplexes which are m-cells but which do not admit transversal $(n-m)$-planes, no matter how they are polyhedrally represented in any E^n. A plane is *transversal* to a point set if it makes angles bounded away from zero with secants of the set. (V) Brouwer's definition of an m-manifold (loc. cit.) is not invariant ($m>3$) under subdivision. For the connection with questions of differentiability, see abstracts 45-3-110 and 45-9-306, the results of which are to appear in the Annals of Mathematics. (Received March 29, 1940.)

356. J. L. Coolidge: *Analytic systems of central conics in space.*

Systems of central conics in three-space have been little studied. Spottiswoode and Johnson have studied algebraic systems and there is a paper by Blutel dealing with surfaces generated by conics of a particular type. In the present paper there is a study of general one- and two-parameter analytic systems. The technique is that of the moving tetrahedron first developed by Darboux and employed in more recent times by Cartan. (Received April 9, 1940.)

357. Richard Courant: *On a generalized form of Plateau's problem.*

From a geometrical viewpoint it is natural to replace the Plateau problem, in which a continuous and monotonic description of the prescribed boundary is required, by another problem where no condition of this type is stipulated. For the solution of this generalized problem, methods quite different from those usually used for Plateau's problem are needed. The main objective of the paper is to show that the solution of the ordinary Plateau problem also solves the new problem, while the converse is not necessarily true. (Received April 11, 1940.)

358. A. González Domínguez: *Some theorems on the Hermite singular kernel.*

Let $g(x)$ be a function of bounded variation in $(-\infty, \infty)$ normalized in the usual way, and let $K(x, y, r)$ be the singular kernel which appears in the Abel summation of Hermite series. Then the following equality holds: $\lim_{r\to 1} \int_0^a dy \int_{-\infty}^{\infty} K(x, y, r)dg(x) = g(a)-g(0)$. This result, and others of a similar character, are used to obtain a uniqueness theorem for Hermite-Stieltjes series, a new limit-theorem for distributions functions, and also to solve the problem of the "classes" for Hermite series. (Received April 4, 1940.)

359. J. E. Eaton: *On the extension of groups.* I. *Normal extensions.* II. *Non-normal extensions.*

I. The problem of finding all groups $\mathfrak{G}$ such that $\mathfrak{G}/\mathfrak{N}\cong\mathfrak{H}$ for given groups $\mathfrak{H}$ and $\mathfrak{N}$ has been considered by Schreier. In this paper Schreier's n^3 associativity conditions on n^2 elements of $\mathfrak{N}$ are replaced by kn more complicated conditions on rn elements, wherein n is the number of elements of $\mathfrak{H}$, k the number of generating relations defining $\mathfrak{H}$, and r the number of generators. The problem of obtaining all non-redundant solutions of these conditions is then considered. II. By using the results of a previous paper (abstract 45-9-310) one may formulate the problem of non-normal extensions of groups: *to determine all groups $\mathfrak{G}$ such that $\mathfrak{G}/\mathfrak{H}\cong\mathfrak{C}$ where $\mathfrak{H}$ is a given group and $\mathfrak{C}$ a given "cogroup."* If one calls those groups $\mathfrak{G}$ for which no proper subgroup of $\mathfrak{H}$ is normal in $\mathfrak{G}$ a *purely non-normal extension*, the problem of non-normal extensions may be reduced to purely non-normal ones followed by normal extensions. Conditions for purely non-normal extensions, analogous to those of Schreier for the normal case, are derived. (Received April 13, 1940.)

360. D. W. Hall: *On arc and tree preserving transformations.*

It is shown that every single valued transformation $T(A)=B$, where A and B are cyclic locally connected continua, which is arc preserving and tree preserving is necessarily a homeomorphism. No assumptions are made as to the continuity of the transformation T. (Received April 26, 1940.)

361. E. R. Lorch: *The integral representation of weakly almost-periodic transformations in reflexive vector spaces.*

As announced previously (abstract 45-11-401), a weakly almost-periodic transformation V in a reflexive space $\mathfrak{B}$ is characterized by the relation $\|V^n f\|\leq K\|f\|$, $n=0, \pm 1, \pm 2, \cdots$. For such a w.a.p. V, an integral representation is given which is the exact counterpart of the known representation $U=\int e^{i\lambda}dE(\lambda)$ for a unitary transformation U in a Hilbert space. Specifically it is proved: I. There exists a family of pairs of closed linear manifolds $\{\mathfrak{E}_\lambda, \mathfrak{F}_\lambda\}$, $-\infty<\lambda<\infty$, with the following properties: (a) $\mathfrak{E}_\lambda$ and $\mathfrak{F}_\lambda$ have only 0 in common; together they span $\mathfrak{B}$. (b) The pair $\{\mathfrak{E}_\lambda, \mathfrak{F}_\lambda\}$ reduces V. (c) $\mathfrak{E}_\lambda\supset\mathfrak{E}_\mu$ for $\lambda>\mu$; $\mathfrak{F}_\lambda\subset\mathfrak{F}_\mu$ for $\lambda>\mu$. (d) $\mathfrak{E}_{-\pi}=0$, $\mathfrak{F}_{-\pi}=\mathfrak{B}$, $\mathfrak{E}_\pi=\mathfrak{B}$, $\mathfrak{F}_\pi=0$. II. Given an $\epsilon>0$, there exists a set of numbers $\lambda_1, \lambda_2, \cdots, \lambda_n$ and closed linear manifolds $\mathfrak{A}_{\lambda_j}$, $j=1, \cdots, n$, such that (a) $\mathfrak{A}_{\lambda_j}$ lies in $\mathfrak{E}_{\lambda_j+\epsilon}$ and in $\mathfrak{F}_{\lambda_j-\epsilon}$, (b) on $\mathfrak{A}_{\lambda_j}$, $\|(V-e^{i\lambda})f\|\leq\epsilon\|f\|$, (c) $\mathfrak{B}$ is spanned by the $\mathfrak{A}_{\lambda_j}$, $j=1, \cdots, n$. III. The spectrum of V is competely characterized by the behavior of $\{\mathfrak{E}_\lambda, \mathfrak{F}_\lambda\}$. That is, for $-\pi<\lambda<\pi$, $e^{i\lambda}$ is in the point spectrum, the continuous spectrum, or the resolvent set of V if and only if λ is a point of discontinuity, of continuity, or of constancy, respectively, of $\{\mathfrak{E}_\lambda, \mathfrak{F}_\lambda\}$. The integral representation and spectral properties of the transformation $H=-i(V-I)(V+I)^{-1}$ are obtained. (Received April 25, 1940.)

362. A. N. Lowan and Jack Laderman: *On the distribution of errors in the computation of tables by differences.*

The values of a function may often be computed conveniently from the leading differences up to the order $r-1$, and the rth differences (which are not assumed to be constant) for all the required arguments. The values obtained by building up the table from the above differences are equal to those obtained from the formula $u_n=u_0+C_{n,1}\Delta u_0+C_{n,2}\Delta^2 u_0+\cdots+C_{n,r-1}\Delta^{r-1}u_0+\sum_{i=0}^{n-r}C_{n-i-1,r-1}\Delta^r u_i$. Expressions are

given for the maximum possible error in u_n, for the distribution of the error, and for the variance of the distribution when the differences have been rounded in the kth decimal place. The above results were obtained in the course of work done by the Project for the Computation of Mathematical Tables, Work Projects Administration, New York City. (Received April 10, 1940.)

363. A. N. Lowan and Hyman Serbin: *On upper bounds for the remainder in the evaluation of differences.*

The nth difference of a function $f(x)$ defined over the interval from x_0 to x_0+nh may be expressed in the elegant form $\Delta^n f(x)=\int_0^h\int_0^h\cdots\int_0^h f^{(n)}(x+\xi_1+\xi_2+\cdots+\xi_n)d\xi_1 d\xi_2\cdots d\xi_n = h^n\int_0^1\int_0^1\cdots\int_0^1 f^{(n)}(x+\xi_1+\xi_2+\cdots+\xi_n)d\xi_1 d\xi_2\cdots d\xi_n$. This leads to $\Delta^n f(x)=h^n f^{(n)}(x)+\sum_{k=1}^{p}(k!)^{-1}h^{n+k}f^{(n+k)}(x)\int_0^1\cdots\int_0^1(\xi_1+\cdots+\xi_n)^k d\xi_1\cdots d\xi_n + [h^{n+p+1}/(p+1)!]\int_0^1\cdots\int_0^1(\xi_1+\cdots+\xi_n)^{p+1}f^{(n+p+1)}(x+\theta(\xi_1+\cdots+\xi_n))d\xi_1\cdots d\xi_n$. If the expression for $\Delta^n f(x)$ is truncated after the term involving the derivative of order $n+p$, an upper bound for the truncating error is given by $[h^{n+p+1}/(p+1)!]\{f^{(n+p+1)}\}\int_0^1\cdots\int_0^1(\xi_1+\cdots+\xi_n)^{p+1}d\xi_1\cdots d\xi_n$ where $\{f^{(n+p+1)}\}$ is an upper bound of $f^{(n+p+1)}(x)$ in the given interval. An upper bound of the last integral is given by n^{p+1}. The application of this formula to x^{n+m} (where m is a positive integer) leads to an expression of Milne-Thomson's Bernoulli numbers $B^{(-n)}$ in terms of the above repeated integrals. The above results were obtained in the course of work done by the Project for the Computation of Mathematical Tables, Work Projects Administration, New York City. (Received April 6, 1940.)

364. M. H. Martin: *Quasi-Lagrangian systems.* Preliminary report.

Non-conservative dynamical systems $L_{\dot q_r}-L_{q_r}+k(t)L_{\dot q_r}=0$ are studied. Such systems have properties allied to Lagrangian systems ($k\equiv 0$) and are termed *quasi-Lagrangian* systems. The restricted problem of three bodies involving the motion of an infinitesimal mass attracted by two finite masses moving in accordance with a preassigned solution of the two body problem leads to a quasi-Lagrangian system, as do certain dynamical systems containing frictional forces. The equations of motion may be put in the variational form $\delta\int_{t_0}^{t_1}e^{kt}L dt=0$, ($\dot l=k$) and in the canonical form $\dot q_r=H_{p_r}$, $\dot p_r=-H_{q_r}-kp_r$ with the partial differential equation $S_t+kS+H(t, q_r, S_{q_s})=0$ playing the role of the Hamilton-Jacobi partial differential equation. The flow in the phase space is not incompressible, the volume V of a region being given in terms of its volume V_0 at $t=t_0$ by the relation $V=e^{-n(t-t_0)}V_0$. When $L_t\equiv 0$ and k is constant there exist generalizations of the energy integral and of the principle of Mapertuis. At least half the characteristic exponents of an equilibrium solution have negative (positive) real parts if $k>0$ ($k<0$). If L contains no linear terms in $\dot q_r$, there are no periodic solutions and positively (negatively) stable motions tend uniformly towards equilibrium points. The nature of flow in the neighborhood of equilibrium points is investigated. (Received April 12, 1940.)

365. H. J. Miser: *Regions and their "patterns" in conformal mapping.*

Let the circle $|z|<1$ be mapped by $w=f(z)$, ($f(0)=0$), on a plane region S, the map of $|z|<r<1$ being S_r. Let $w_0=T(w)$ be analytic in S, and let $T(0)=0$. If w_0 is in S for all w in S, then S is said to have the property T. It has been proved (L. R. Ford, Duke Mathematical Journal, vol. 1 (1935), pp. 103–104) that if S has the property T, so also does S_r. This paper considers the function $w_0=tw$, and studies the set

of values Σ in the t-plane, which is called the "pattern" of S, for which S has the property T. The set Σ contains 0 and 1, lies in $|t| \leq 1$, is closed, and is contained in the pattern Σ_r of S_r. (Received April 5, 1940.)

366. I. E. Segal: *Certain topological rings.*

The rings considered are Banach spaces which are rings, with the norm having the property $|ab| \leq |a| \cdot |b|$, for example, the ring of all Fourier-Stieltjes transforms. Theorems are derived which connect the properties of an element modulo the closed maximal ideals of the ring with the existence of an inverse to the element, or the existence of any given analytic function of the element. Some special cases are treated. (Received March 29, 1940.)

367. E. W. Titt: *On the characteristic theory of a first order partial differential equation.*

This paper can be regarded as a continuation of a previous study of the relation between the $(n-1)$-dimensional and the one-dimensional characteristic manifolds of a first order partial differential equation (Duke Mathematical Journal, vol. 3 (1937), p. 740). The method is as follows: first, derive the differential equations of an $(n-1)$-dimensional characteristic manifold as a set of consistency conditions which must be satisfied over exceptional manifolds, that is, manifolds over which the determinant of the left member of a system of linear equations vanishes; regard this set of consistency conditions as a new system of differential equations to be solved; repeat the characteristic process on this system of equations to obtain characteristic sub-manifolds of $(n-2)$ dimensions. Continuing this step-by-step process we finally arrive at characteristic sub-manifolds of one dimension. These are shown to be the classical characteristic strips of Cauchy and Lie. (Received March 29, 1940.)

368. A. D. Wallace: *On separation spaces.*

The purpose of this paper is to give a new system of axioms for topological spaces in which the notion of "mutual separation" is taken as undefined. It is shown that a set of separation axioms suffice to characterize a T_1-space, but examples indicate that a weaker topology would be more natural. Separation spaces seem to be well adapted to the study of analytic topology and a version of the Whyburn-Eilenberg factor theorem is given under very general conditions and without the use of upper semi-continuous decompositions. (Received April 26, 1940.)

369. J. L. Walsh and W. E. Sewell: *Degree of polynomial approximation to analytic functions. Problem β.*

If $f(z)$ is analytic for $|z| < 1$ and satisfies the inequality $|f(z)| \leq M_1(1-|z|)^{p+\alpha}$, where p is a negative integer and $0 < \alpha < 1$, $f(z)$ is said to be of class $L(p, \alpha)$. If $f^{(p)}(z)$ is analytic for $|z| < 1$, continuous for $|z| \leq 1$, and satisfies a Lipschitz condition on $|z| = 1$ of order α, where p is a nonnegative integer and $0 < \alpha < 1$, $f(z)$ is said to be of class $L(p, \alpha)$. If $f(z)$ is of class $L(p, \alpha)$, there exist polynomials of respective degrees n such that $|f(z) - p_n(z)| \leq M\rho^{-n}n^{-p-\alpha}$ on the circle $|z| = 1/\rho < 1$. Reciprocally if there exist polynomials of respective degrees n such that $|f(z) - p_n(z)| \leq M\rho^{-n}n^{-p-\alpha-1}$ on the circle $|z| = 1/\rho$, then $f(z)$ is of class $L(p, \alpha)$. These results extend to approximation on an arbitrary analytic Jordan curve. (Received April 15, 1940.)

370. J. V. Wehausen: *A space isomorphic to Hilbert space.*

A Banach space is constructed which is isomorphic to Hilbert space but which

contains no euclidean spaces of dimension equal to or greater than 2. It is constructed as follows. Let $x=(x_1, x_2, \cdots)$ and $x^{(k)}=(x_1, x_2, \cdots, x_k, 0, \cdots)$. Define $\|x^{(1)}\|=|x_1|$, $\|x^{(k+1)}\|_{p_k}=\|x^{(k)}\|_{p_k}+|x_{k+1}|_{p_k}$, $(p_k \geqq 1)$, and $\|x\|=\lim_{k\to\infty}\|x^{(k)}\|$. The set of sequences for which $\|x\|$ is finite is the required space if p_k converges to 2 sufficiently fast ($p_k=2-1/k$ is adequate). (Received April 10, 1940.)

371. G. T. Whyburn: *On multicoherence.*

Consider the number $r(X)$ defined by Eilenberg for continua X as l.u.b. $[r(X_1, X_2)]$, where $X=X_1+X_2$ is any decomposition of X into continua X_1 and X_2 and $r(X_1, X_2)+1$ is the number of components of $X_1 \cdot X_2$. It is shown that this number is cyclicly additive, that is, $r(M)=\sum r(E_i)$, where M is a locally connected continuum with true cyclic elements $[E_i]$. Also a new and direct proof (based on the simple fact that if a sum of k connected sets is connected, a connected partial sum consisting of any given number less than or equal to k of terms can be found) is given for the result due essentially to Eilenberg that if the transformation $f(A)=B$ is quasi-monotone in the sense of Wallace (that is, for each continuum K in B with a non-vacuous interior, $f^{-1}(K)$ consists of a finite number of components each mapping onto K), $r(B) \leqq r(A)$. This gives $r(B) \leqq r(A)$ for monotone transformations on continua A and for interior transformations on locally connected continua A. (Received April 27, 1940.)

372. G. T. Whyburn: *On singularities and inversion under interior transformations.*

A subset K is said to locally separate a set M at $x \in K$ provided that for every sufficiently small neighborhood V of x in M, $V-V \cdot K$ is disconnected. It is shown that if A is locally compact, $f(A)=B$ is continuous and interior on $A-E$ where $E \subset A$, x is any point of $E \cdot \overline{A-E}$ which is a component of $f^{-1}f(x)$, and U is any neighborhood of x, then $f(E \cdot U)$ locally separates B at $f(x)$. Thus, in particular, if E is not open in A and $f(E)$ does not locally separate B at any point, f is interior on A. (This includes and extends results of Stoilow and the author on singularities of interior transformations.) It is also shown that, if A is a locally connected continuum and $f(A)=B$ is interior and light, (1) if Y is a locally connected continuum in B whose interior is dense in Y, $f^{-1}(Y)$ is locally connected, (2) if A is on a 2-manifold, and p is any point of B of order greater than 2, $f^{-1}(p)$ is a finite set of points. These two results form the basis for a greatly simplified proof for the theorem that, under the conditions of (2), if K is a locally connected continuum in B, $f^{-1}(K)$ is locally connected. (Received April 27, 1940.)

373. G. T. Whyburn: *Transformations on boundary curves.*

For any subset $\overline{X}$ of a continuum M let $\alpha(\overline{X})$ denote the number of components of $M-\overline{X}$. If A is a boundary curve (equal to a Peano continuum every true cyclic element of which is a simple closed curve) and $\underline{P}=p_1+p_2+\cdots+p_m$ is any finite subset of M, it is shown that $\alpha(\underline{P}) \leqq \alpha(p_1)+\cdots+\alpha(p_m)$ and the equality holds if, for each i and j, $\underline{P}$ contains all points of A separating p_i and p_j. This is used to prove that if $f(A)=B$ is non-alternating and light, for each $y \in B$, $\alpha(y)=\sum\alpha(x)$, $x \in f^{-1}(y)$. The converse holds if B is assumed to be a boundary curve or if $\alpha(y)$ is assumed finite for every $y \in B$. It results from this that if A, B and C are boundary curves and the transformations $f_1(A)=B$ and $f_2(B)=C$ are non-alternating and light, so also is the composite transformation $f(A)=f_2f_1(A)=C$. (Received April 27, 1940.)

374. Paul Alexandroff: *General combinatorial topology.*

The fundamental duality theorem which correlates the r-dimensional cohomology group of a closed set with the $(r+1)$-dimensional cohomology group of the residual set is here obtained by a systematic development of homology theory based on the inverse and direct spectra. The same development leads also to a simple definition of the cocycle ring. Thus the theory of the spectrum, a standard topological and group-theoretical concept, is now made to include results which have hitherto been obtained only by the special methods of Alexander and of Kolmogoroff. (Received May 8, 1940.)

375. J. P. Ballantine: *Proof of Poincaré's geometric ring theorem.*

Referring to the proof of Poincaré's geometric ring theorem (Transactions of this Society, vol. 14 (1913)) by G. D. Birkhoff, the author has attempted to supplement the reasoning given there by two lemmas. It is assumed that T is a continuous one-to-one transformation having no invariant point. The index of a closed curve C relative to T is defined as the number of revolutions advanced by the vector from P to $T(P)$ as P moves around C. Lemma 1: If there exists a circuit of index different from zero relative to T, there is one of index different from zero which can be inscribed in a circle of radius ϵ. Lemma 2: If there exists a circuit C of index different from zero inscribed in a circle of radius ϵ, and if $2\epsilon<\delta$, where δ is the least distance moved by any point under T, then the index of C is ± 1, $T(C)$ surrounds the circle of radius ϵ, and the area of $T(C)$ exceeds that of C. It follows that T is not area preserving. (Received June 24, 1940.)

376. E. F. Beckenbach: *An integral analogue of Laplace's equation.*

It is shown that if the real function $x(u, v)$ is continuous in a finite simply connected domain D then a necessary and sufficient condition that $x(u, v)$ be harmonic in D is that $\int_{C(u_0, v_0; r)}\left[\int_{C(u_0+s, v_0+t; \rho)} x(u_0+s+\xi, v_0+t+\eta)(d\xi+id\eta)\right](ds-idt)=0$ hold for all circles $C(u_0, v_0; r+\rho)$ in D. (Received April 27, 1940.)

377. R. S. Burington and J. M. Dobbie: *A new family of wing profiles.*

Piercy, Piper, and Preston (Philosophical Magazine, (7), vol. 24 (1937), pp. 425–444, [P]), using conformal mapping, have recently advanced a new family of wing profiles F which are thought to have certain advantages over earlier theoretical wing shapes. The present paper reconsiders this work from a rigorous mathematical point of view, with the purpose of definitely exhibiting a transformation which fulfills all the requirements as to analyticity and boundary conditions essential to the theory (Burington, *On the use of conformal mapping in shaping wing profiles*, to appear in the American Mathematical Monthly). Special attention is given to the singularities of the transformations used. A direct by-product of this study leads to a new and more general family B of airfoils, a subfamily of which is the family F. The sequence of transformations advanced herein, which maps a circle into an airfoil of family B (and F), seems much simpler than that used in the paper [P]. (Received May 15, 1940.)

378. J. H. Curtiss: *Necessary conditions in the theory of interpolation in the complex domain.*

Let $f(z)$ be a function which is analytic on a closed limited region R whose boundary B is the boundary of an infinite region K. Let $L_n(z; f)$ be the polynomial of degree at most $n-1$ which coincides with $f(z)$ in the points $\alpha_1^{(n)}, \alpha_2^{(n)}, \cdots, \alpha_n^{(n)}$ chosen on B. If at a single interior point z_0 of R, $\limsup_{n\to\infty} |L_n[z_0; 1/(t-z_0)]|^{1/n} \leqq 1$, all t in K, it follows that (i) $|\omega_n(z)|^{1/n} \to \Delta$ uniformly on any closed point set of the interior R, where Δ is the transfinite diameter of R and $\omega_n(z) = \prod_1^n (z-\alpha_k^{(n)})$, (ii) $L_n(z; f) \to f(z)$ uniformly, z in R, for each function $f(z)$ analytic on R, (iii) if B is a Jordan curve, the points $\alpha_k^{(n)}$ are uniformly densely distributed on B. In establishing this result, several auxiliary results are developed in some detail, including some new non-trivial sufficient conditions for convergence of the sequence $\{L_n\}$ and an inequality which promises to be useful in the study of the polynomials $\omega_n(z)$. (Received May 31, 1940.)

379. E. L. Dodd: *The probabilities for certain inequalities among the values of a moving average of chance variables.*

Let independent chance variables x_i be subject to the same law of distribution. Let $z_r = \sum x_i$, $i = r-n+1$ to r. Let p be the probability that $z_r \leqq z_{r+1} \leqq \cdots \leqq z_{r+k}$ or, more generally, that among $k+1$ consecutive z's a specified ordering should obtain. Then for every $n \geqq k$, this value of p is a constant. If, further, the x's are normally distributed, and s and t are whole numbers, the probability that $z_{r-s} \leqq z_r \geqq z_{r+t}$ is at least equal to $\frac{1}{4}$. The above statements remain valid if z_r denotes the moving average $\sum x_i/n$, instead of the moving sum. (Received May 20, 1940.)

380. H. E. Goheen: *On the primitive groups of the classes $4p$ and $5p$.*

The author proves that there are no primitive groups of class $5p$, p a prime greater than 5, which contain a permutation of degree $5p$ and order p, and that with a few exceptions there are no primitive groups of class $4p$ which contain a permutation of order p and degree $4p$. The exceptions are $p=5$, in which case every such group is contained in the holomorph of the elementary abelian group of order 25, and $4p+1=q$, q a prime or a power of 5, in which case all the known groups of the required type are primitive subgroups of the triply transitive Mathieu group. The proof is based upon the examination of the normalizer of the cyclic group generated by the permutation of order p and minimum degree in the group in the light of a theorem of W. A. Manning (this Bulletin, vol. 13 (1906), p. 201), and the examination of the subgroup fixing one letter of a simply transitive primitive group in the light of a theorem of Weiss (this Bulletin, vol. 40 (1934), p. 402). Together with auxiliary considerations these dispose of every possibility except one, which is disposed of by means of the theory of group characters. (Received May 22, 1940.)

381. O. G. Harrold (National Research Fellow): *A note on strongly irreducible maps of an interval.*

A continuous mapping of the compact metric space A onto B has been called *strongly irreducible* provided no proper closed subset of A maps onto all of B. If M is any Peano space in which the nonlocal separating points are dense and P is any given countable dense set of nonlocal separating points, there exists a continuous mapping, $f(I) = M$, where I denotes the unit interval, such that $y \in P$ implies $f^{-1}(y)$ is

a single point and $\overline{f^{-1}(P)} \supset I$. The set J^* of such strongly irreducible maps is a dense G_δ in the space J of maps of I onto M. These mappings are nowhere arc preserving in the sense that the only arcs which are transformed into arcs are the individual points. (Received May 21, 1940.)

382. L. B. Hedge: *Moment problem for a bounded region.*

A solution of the moment problem for a bounded interval, given by Hausdorff in 1923, is extended to any bounded region in euclidean n-space under certain conditions on polynomial expansions over the region. The resulting solution is shown to be valid for the n-dimensional sphere, and includes the Hausdorff case as well as known conditions on the "class" of Fourier and Fourier-Stieltjes series. (Received May 17, 1940.)

383. Ralph Hull: *The structure of certain Fuchsian groups.*

The units of a maximal order of a rational, indefinite, quaternion algebra form an infinite group which may be represented as a Fuchsian group, and whose structure is determined in a simple way by the fundamental number of the algebra (cf. American Journal of Mathematics, vol. 61 (1939), pp. 365–374). The group of units of a non-maximal order of such an algebra may similarly be represented as a Fuchsian group, and analogous methods may be applied to determine its structure, that is, the genus, and class-number of elliptic substitutions in the group. Groups thus studied include those which correspond to the totality of integral solutions of equations such as $x^2+ay^2-bu^2-abv^2=1$, where a and b are positive integers. (Received May 21, 1940.)

384. Bella Manel: *Conformal mapping of multiply connected domains on the basis of Plateau's problem.*

This paper is concerned with the conformal mapping of an arbitrary multiply connected plane domain G on the following normalized domains: B_1, the half-plane with finite slits radial to the origin one of which lies on the imaginary axis; B_2, the half-plane with elliptical holes having their axes mutually parallel and having a fixed eccentricity; B_3, the unit circle with concentric circular slits one of which has end points on the real axis. The method is to solve the Plateau problem for the contour bounding G with the desired conformally equivalent domains as the domains of representation. The solution yields the analytic function which performs the mapping. This connection between conformal mapping and the Plateau problem has been stressed by Douglas and by Courant. The procedure of Courant forms the basis of this investigation. For plane contours the existence of a solution to a minimum problem involving the Dirichlet integral is used. By a detailed analysis of the variational conditions it is shown that the solution to this minimum problem is a minimal surface for each of the normal domains B_1, B_2, B_3. (Received May 27, 1940.)

385. A. F. Moursund: *A note on Gibbs' phenomenon.*

The kernel for Nevanlinna's weak summation method, which is essentially the same as the Bosanquet-Linfoot zero order method, contains a constant whose value has no effect in determining the strength of the method in forcing convergence. In this note it is shown that this constant plays an important role in determining the power of the method to destroy Gibbs' phenomenon in Fourier series. Weak methods are exhibited which destroy Gibbs' phenomenon while stronger methods do not. (Received May 22, 1940.)

386. T. S. Peterson: *The Fredholm minors for Goursat's kernel.*

The purpose of this note is to give the explicit determinantal form for the Fredholm minors of a kernel having the form $\sum_{i=1}^{n} M_i(x)N_i(y)$. The result is obtained directly using theorems due to Platrier and Sylvester. (Received May 21, 1940.)

387. R. M. Robinson: *On the approximation of irrational numbers by fractions with odd or even terms.*

All fractions may be divided into four types, according as the numerator and denominator are odd or even. Let one or more of these types be selected as admissible. Using continued fractions, the largest value of μ is determined for which to every irrational ξ infinitely many admissible fractions A/B can be found satisfying $|\xi - A/B| < 1/\mu B^2$. A part of this problem was solved by W. T. Scott in the February issue of this Bulletin, using a different method. (Received May 29, 1940.)

388. P. M. Swingle: *Local connectedness and biconnected sets.*

In this paper it is proved for certain spaces that if a biconnected set B is connected im kleinen at a point p, then p is a dispersion point of B. The problem is generalized in several ways. (Received May 17, 1940.)

389. P. M. Swingle: *Non-transfinite connected sets.*

In this paper the definitions of finitely-containing, finitely-divisible, and finitely-convergeable connected sets are modified, substituting in place of connected subset either biconnected, widely connected, or punctiform connected subset. The relations between these new types of sets are studied, a solution being obtained of previously proposed problems (American Journal of Mathematics, vol. 53, pp. 373, 400, Problems 3 and last half 11). Also certain results are obtained concerning a modular connectivity which is defined. (Received May 17, 1940.)

390. C. W. Vickery: *On cyclically invariant graduation.* II.

The author has extended the results of an earlier paper (abstract 46-5-336, April 27, 1940). Suppose that $g(t)$ is a periodic function of period $2k$ having an expansion in Fourier series. Suppose that X is a random variable with characteristic function $f(w)$ such that, for every integer n, $f(n\pi/k) \neq 0$ and is an even function. Let $g^*(t) = g(t) - a_0/2$, where $a_0 = (1/k)\int_{-k}^{k} g(t)dt$. For every integer $m \geqq 0$, graduation operator G_m is defined as follows: $G_m\{g(t)\} = a_0/2 + E\{g^*(t+X)\}/f(m\pi/k)$. The terms of index m of the Fourier expansion of $g(t)$ are invariant with respect to the application of G_m. Necessary and sufficient conditions have been found that the application of G_m to any Fourier series produce a Fourier series. Sufficient conditions have been found that the application of G_m to any Fourier series produce a Fourier series that is convergent almost everywhere. (Received April 23, 1940.)

391. C. W. Vickery: *On systems of Fourier coefficients.*

If $\sum (a_n \cos nx + b_n \sin nx)$ is a Fourier series and $[\lambda_n]$ is a sequence of real numbers, then in order that $\sum \lambda_n(a_n \cos nx + b_n \sin nx)$ be a Fourier series it is necessary and sufficient that there exist a periodic function $g(x)$ of bounded variation and of period 2π such that, for each integer m, $f(m) = 1/\pi \int_{-\pi}^{\pi} e^{imx} dg(x)$ is an even function and such that $\lambda_n = f(n)$, n being positive. An example shows that it is not necessary

that $g(x)$ be an odd function. If, furthermore, the second derivative of $g(x)$ exists for all values of x between $-\pi$ and π and is absolutely integrable, then $\sum\lambda_n(a_n \cos nx + b_n \sin nx)$ is a Fourier series which converges for all values of x except a set of measure 0. (Received April 23, 1940.)

392. H. S. Wall: *A continued fraction related to partition formulas.*

The author shows how four well known identities involving $\prod(1-ax^n)$ can be derived from a single continued fraction, namely, $1+rx/1+(1-r)wx/1+(1-w)wrx/1+(1-wr)w^2x/1+(1-w^2)w^2rx/1+\cdots$. (Received April 6, 1940.)

393. Y. K. Wong: *On biorthogonal matrices.*

Consider the basis $\mathfrak{A}$, $\mathfrak{P}^1$, $\mathfrak{P}^2$, ϵ^1, ϵ^2 (E. H. Moore, *General Analysis*, Part I, p. 16). Suppose that κ^{12} is by columns of $\mathfrak{M}(\epsilon^1)$. In a paper entitled *On non-modular matrices*, the author establishes the one-to-one correspondence between the class $\mathfrak{M}(\epsilon_\kappa^1\ \kappa^*\ \epsilon^2)$ of all vectors μ^1 modular as to ϵ^1 such that $J^1\kappa^{*21}\mu^1$ are in $\mathfrak{M}(\epsilon^2)$ and the class $\mathfrak{M}(\epsilon^2 \cap \xi_\kappa^2)$ of vectors μ^2 modular as to ϵ^2 and ξ_κ^2. In the present paper, the author first obtains some properties on a pair of matrices κ^{12}, ψ^{21}, which are by columns of $\mathfrak{M}(\epsilon^1)$, $\mathfrak{M}(\epsilon^2)$ respectively, such that $J^2\kappa^{*12}\mu^2 = J^2\psi^{*12}\mu^2$ for every μ^2 in $\mathfrak{M}(\epsilon^2 \cap \xi_\psi^2)$. Next, it is assumed that ϵ_0^1, ϵ_1^1 are idempotent as to ϵ^1, κ^{12} is by columns of $\mathfrak{M}(\epsilon_0^1)$, and ϕ^{21} is by rows conjugate of $\mathfrak{M}(\epsilon_1^1)$. Take any pair of vectors μ^1, ν^1 in $\mathfrak{M}(\epsilon_0^1\ \kappa^*\ \epsilon^2)$, $\mathfrak{M}(\epsilon_1^1\ \phi\ \epsilon^2)$ respectively; if $J^1\bar{\mu}^1\nu^1$ is equal to $J^2(J^1\bar{\mu}^1\kappa^{12}, J^1\phi^{21}\nu^1)$, then ϕ^{21}, κ^{12} are said to be biorthogonal as to $\epsilon^2\epsilon_1^1\epsilon_0^1$. The purpose of the paper is to investigate the properties of the biorthogonal pairs of matrices. (Received May 22, 1940.)

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